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ABSTRACT
The nonlinear dynamics of plasma wakefield in the interaction between a relativistic Gaussian electron beam and an inhomogeneous plasma is theoretically studied. The effects of physical parameters, such as the length of the driving electron bunch, the initial plasma density profile, and the static magnetic field strength on the evolution of the plasma wakefield amplitude, are discussed. It is found that the amplitudes of both the longitudinal electric field and the perturbed electron density behind the beam are larger in an inhomogeneous plasma than in a homogeneous plasma. Moreover, in a medium with periodical density variations, the change in the plasma wakefield amplitude is periodical and so the perturbed density behind the beam. It is also found that the plasma wakefield is maximum for a definite length of the drive bunch along the propagation direction. Thus, for a special system of plasmas and drive bunches, it is possible to transfer the maximum energy from a driving electron beam to the particles in the witness bunch and accelerate them to higher energies.

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I. INTRODUCTION

The plasma wakefield acceleration (PWFA) is a standard procedure for accelerating charged particles to relativistic energies. In the 1970s, Tajima and Dawson suggested the idea of propagating high-intensity short laser pulses through a plasma to accelerate the charged particles. They showed that a high-intensity short laser pulse can create a strong longitudinal electric field and proved that the generated wakefield in this system can accelerate electrons to several GeV/cm. Amiranoff et al. were the first group who reported laser wakefield acceleration (LWFA) of electrons in laboratory plasmas. In 1985, a new scheme was presented based on plasma wakefield acceleration (PWFA) for accelerating electrons. Additionally, they were shown that the interaction of a relativistic Gaussian electron beam with a cold unmagnetized plasma can accelerate the electrons with an energy gradient exceeding 1 GeV/m.

The physical mechanism of plasma wakefield excited by a short laser pulse or charged particle bunch is very simple. When a high-intensity short laser pulse or charged particle bunch passes through an underdense plasma, it can push away the plasma electrons from their equilibrium position and these perturbed electrons begin to oscillate due to the ponderomotive force of the laser pulse or the charge-space force of the charged particle bunches. These oscillations have a waveform to follow their driver source with a phase velocity that is equal to the group velocity of the driver source. Because the plasma is an ionized medium, it can support a large number of oscillation modes (electric fields) without being destroyed. Therefore, we can utilize them for accelerating charged particles to relativistic energies. Indeed, the plasma density must be greater than $10^{14}$ cm$^{-3}$ for obtaining accelerating fields of the order of GV/m in PWFA. Therefore, we have to recognize plasmas as strong accelerators for accelerating electrons. In PWFA and LWFA methods, many valuable research works have been conducted in both theoretical and experimental aspects. One of the ways to study plasma wakefields is to investigate the plasma or driving source parameters that can affect wakefields. It has been
found that inhomogeneity in the plasma density profile and the chirp shape of a laser pulse can increase the amplitude of a plasma wakefield and so the acceleration of the plasma electrons in LWFA.\(^1\)\(^2\)\(^3\) The interaction of a Bessel–Gaussian laser pulse with a homogenous plasma can generate a stronger plasma wakefield compared to a Gaussian laser pulse.\(^1\)\(^2\)\(^4\)\(^5\) In the lab environment, it has been found that due to the interaction between a high-intensity short laser pulse or an electron beam and a plasma medium, a high quality electron beam can be generated.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\)

One of the important parameters that can control the wakefield amplitude in a plasma medium is the density profile of the background electrons. Both the maximum energy gain and the acceleration of particles in a plasma can be controlled by the plasma density profile.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\) The frequency and duration of the trailing microbunches can be controlled by the plasma density gradient and driving beam.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\) A train of short charged particle bunches can excite a strong wakefield in the plasma if all bunches reside in focusing and decelerating phases of the wakefield.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\) Applying an external magnetic field to a homogeneous plasma can play an important role in the value of wakefield amplitude.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\) Therefore, the external magnetic field can affect the transformer ratio, i.e., the maximum energy gained by a witness bunch to the maximum energy loss of a drive bunch or a laser pulse. Actually, the acceleration of charged particles by a plasma wakefield can be verified in many situations, such as in laboratory and astrophysical plasmas. Wakefield excitation in plasmas surrounding the pulsars is performed by the propagation of charged particle bunches.\(^1\)\(^2\)\(^3\)\(^4\)\(^5\)

In this article, we consider a relativistic Gaussian electron beam that propagates in a cold inhomogeneous magnetized plasma system and generates a wakefield. The effect of the inhomogeneity of the background plasma electron density on the excitation of the plasma wakefield is studied. Moreover, the excitation of the plasma wakefield and amplitude of the perturbed electron density behind the beam are compared for two different background electron densities in the presence of a constant external magnetic field. The plasma wakefield amplitude variations depend on the variation factor of inhomogeneity. The effect of variation in the drive bunch length along the propagation direction on the amplitude of plasma wakefield is also investigated.

This paper is organized into four sections. In Sec. II, the dynamics of the beam–plasma interaction that we use to describe the excitation of the nonlinear plasma wakefield are presented by using Maxwell equations and the continuity equation. In Sec. III, we present the numerical solution of ordinary differential equations (ODEs), which describe the excitation of wake waves and perturbed electrons behind the driving beam. Finally, the results and conclusion of the research are given in Sec. IV.

II. DYNAMICS OF BEAM–PLASMA INTERACTION

Here, we present the general designation of the electron dynamics in the beam–plasma interaction. When an electron bunch moves through a cold inhomogeneous magnetized plasma, the electron continuity equation, the equation of motion, and Maxwell equations describe this motion. It is considered that the electron beam propagates along the x-direction in the plasma. Accordingly, the continuity equation can be written as follows:

\[
\frac{\partial n_b}{\partial t} + \frac{\partial}{\partial x} (n_b v_b) = 0, \tag{1}
\]

where \(v_b\) and \(n_b\) are, respectively, the velocity and density of the plasma electrons. To analyze the electron dynamics, in this case, we use the three-dimensional equation of motion. It is also assumed that the electric field, \(\vec{E}\), is parallel to the direction of the electron beam propagation and the external magnetic field is perpendicular to this electric field in the z-direction. Therefore, we can write the equation of motion as

\[
\frac{\partial \vec{P}}{\partial t} = -e [\vec{E} + \frac{\vec{v}_b}{c} \times (\vec{B} + \vec{B}_0)], \tag{2}
\]

where \(e\) and \(c\) are the elementary charge and speed of light. In addition, \(\vec{P} = p_i \hat{i} + p_j \hat{j} + p_k \hat{k}\) is the plasma electron momentum and \(\vec{B}\) and \(\vec{B}_0\) are the intrinsic and the external magnetic fields, respectively. In addition, it is supposed that \(\vec{B} \ll \vec{B}_0\). Finally, the Maxwell equations in the beam–plasma system can be written as follows:

\[
\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \tag{3}
\]

\[
\nabla \cdot \vec{E} = -4\pi ne (n_s + n_b - n_i), \tag{4}
\]

\[
\nabla \times \vec{B} = \frac{-4\pi}{c} (-n_e \vec{v}_b + \vec{j}_b) + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}. \tag{5}
\]

In Eqs. (3)–(5), \(\vec{j}_b = -n_e \vec{v}_b\) is the beam current density, with \(n_b\) as the density of the charged particle bunch, \(v_b\) as the beam velocity in the x-direction, and \(n_i\) as the density of background ions. We assume that the ions remain immobile and their density is constant. We use an underdense Gaussian beam density profile as

\[
n_b = n_{b0} \exp \left[ -\frac{(\xi - \xi_p)^2}{\sigma_x^2} \right], \tag{6}
\]

where \(\xi_p\) and \(\sigma_x\) are, respectively, the coordinates of the electron beam peak and bunch length along the x-axis that in this paper its value is considered to be 1.9; \(n_0\) and \(n_{b0}\) are the densities normalized to \(n_0\). Note that other variables have their common meanings. The nonrelativistic wave-breaking field can be obtained from Gauss’s law, i.e., \(E_{WB} = n_0 e c / \epsilon_0\), where \(\epsilon_0 = \sqrt{4\pi n_e e^2} / m_e\) denotes the background electron plasma frequency. Therefore, the plasmas can sustain wake waves with longitudinal electric fields of the order of \(GV/m\), or

\[
E_{WB} = \frac{m_e c^2}{\epsilon_0} \sqrt{n_e} \approx 100 \sqrt{n_e} (cm^{-3}) \left( \frac{V}{m} \right), \tag{7}
\]

where \(m_e\) and \(\epsilon_0\) are the electron mass and the vacuum permittivity, respectively. According to Eq. (7), we expect that the generated longitudinal component of the electric field in the plasma is proportional to the local electron density of the plasma, and for an accelerating electric field of the order of \(GV/m\), we should have a plasma medium with \(n_e \geq 10^{14} cm^{-3}\). The plasma electron density \(n_e\) and plasma wavelength \(\lambda_{pe}\) are related to each other by

\[
\lambda_{pe} = \sqrt{\frac{m_e c^2 \epsilon_0}{n_e e^2}}, \tag{8}
\]
Hence, according to the above equation for electron density is mentioned, the plasma wavelength should be about a few millimeters, and according to $\sigma_0 = \lambda_p/\sqrt{2\pi}$, for satisfying the resonance condition, we need a drive bunch with a few hundred micrometers in length along the propagation direction. Because we considered that the electron beam propagates in the $x$-direction, it is convenient to introduce the accompanying frame of reference $\xi$ to perform numerical simulations of the interaction between the ultra-relativistic charged particle beam and plasma, where $\xi = k_p(x - v_{ge}t)$ is an argument, and $k_p = \omega_{pe}/v_{ge}$, where $v_{ge}$ is the group velocity. It is convenient to rewrite the plasma frequency as normalized; therefore, we have
\[
\omega_{pe} = \omega_{pe0}\sqrt{n_i/n_0},
\] (9)
where $\omega_{pe0} = \sqrt{4\pi n_{e0}e^2/m_e}$, with $n_{e0}$ as the initial electron density that is equal to the ion density. Now, in order to perform numerical simulations, we substitute the argument $\xi$ in the equation of motion. Maxwell equations, and continuity equation and employ the following normalization: $P_x \rightarrow P_x/m_e c$, $P_y \rightarrow P_y/m_e c$, $n \rightarrow n_i/n_0$, and $E_W \rightarrow E_i/E_{WB}$. Accordingly, it is replaced with the new variables $v_y/c \rightarrow \beta_y$ and $v_y/c \rightarrow \beta_y$, which are, respectively, the dimensionless velocities of electrons and beams normalized to the speed of light. Therefore, the following equations are obtained:
\[
\sqrt{n_0}\left[1 - \frac{P_x}{\beta_x}\frac{1 + P_x^2 + P_y^2}{\sqrt{1 + P_x^2 + P_y^2}}\right] \frac{\partial P_x}{\partial x} - \frac{\Omega P_y}{\sqrt{1 + P_x^2 + P_y^2}} = E_W = 0,
\] (10)
\[
\sqrt{n_0}\left[1 - \frac{P_x}{\beta_x}\frac{1 + P_x^2 + P_y^2}{\sqrt{1 + P_x^2 + P_y^2}}\right] \frac{\partial P_y}{\partial x} + \frac{\Omega P_x}{\sqrt{1 + P_x^2 + P_y^2}} = 0,
\] (11)
\[
\sqrt{n_0}\left[1 - \frac{n_0}{\beta_x}\frac{1 + P_x^2 + P_y^2}{\sqrt{1 + P_x^2 + P_y^2}}\right] \frac{\partial n}{\partial x} + \eta \beta_x = 0,
\] (12)
\[
\frac{\partial}{\partial x}\left[n(1 - \frac{\beta_y}{\beta_y})\right] = 0.
\] (13)

In Eqs. 10–13, $n = n_i/m_i$ is the normalized electron beam density and $\Omega = \omega_c/\omega_{pe0}$ is the normalized cyclotron frequency, where $\omega_c = eB_0/m_e c$ is the electron cyclotron frequency. In addition, $n$ is the perturbed density of electrons behind the beam, which is normalized to the density of background ions.

III. RESULTS AND DISCUSSION

Here, we study the numerical simulation of the relativistic Gaussian electron beam in a cold inhomogeneous magnetized plasma. Equations (10)–(13) are solved numerically using the 4th order Runge-Kutta method to generate plasma wakefields. It is known that the phase velocity of a wake wave is equal to the group velocity of the electron beam; here, it is assumed that the phase velocity is very close to the speed of light and almost constant. We compare two plasma systems as the backgrounds through which the electron beam passes. These systems have variable densities in the direction of the $x$-axis, as expressed in Eqs. (14) and (15).

For the first system, the plasma electron density variation is expressed as follows:
\[
n(x) = (1 - \chi) \frac{1 + \cos(2\pi x/L)}{2}, \quad -L \leq x \leq L,
\] (14)
where $n(x) = n_i/n_0$ is the electron density normalized to the ion density ($n_i$) and $\chi$ is the variation factor of inhomogeneity over the interval ramp. It is considered that the density varies by a factor of one-quarter ($\chi = 0.25$). For the second medium, the plasma electron density variation is expressed as follows:
\[
n(x) = 1 + bx.
\] (15)

In the above equation, the second term concerns the contribution of the initial density of electrons that due to normalization is one and $b = (n_i - 1)/2L$, where $L$ is the half-length of the plasma medium along the $x$-axis.

In the first step, we assume that the plasma is homogeneous and magnetized. The background density of electrons, the profile of the relativistic Gaussian electron beam, the stationary longitudinal electric field component, and the perturbed density profile behind the beam are shown in Fig. 1. In Fig. 1, the sawtooth-like structure of a plasma wakefield and electron density spikes are observed.

A. Cosine density profile

Let us consider the interaction of a relativistic Gaussian electron beam with a cold magnetized inhomogeneous plasma. The profile of density variation is described in Eq. (14). Variation of background electrons, the profile of the relativistic Gaussian electron beam, also, the evolution of plasma wakefield, and the perturbed electron density behind the beam, for this case, are shown in Fig. 2. The presence of inhomogeneity can affect the nature of these stationary structures, and consequently, the values of plasma wakefield and perturbed electron density amplitude change in this medium. It is evident from Fig. 2 that the amplitude of the normalized longitudinal

![FIG. 1. Background density of electrons, the profile of a relativistic Gaussian electron beam, the longitudinal normalized electric field, and the amplitude of normalized perturbed electron density behind the beam in a cold homogeneous magnetized plasma. The beam parameters are $\beta_0 = 0.5$, $\xi_p = 3$, $\alpha = 1.9$, $\beta_y = 0.995$, and $\Omega = 0.2$.](scitation.org/journal/adv)
FIG. 2. Background density of electrons, the profile of a relativistic Gaussian electron beam, the longitudinal normalized electric field, and the amplitude of normalized perturbed electron density behind the beam in a cold inhomogeneous magnetized plasma with a periodical density profile. The beam parameters are $n_b^0 = 0.5$, $\xi_p = 3$, $\sigma_x = 1.9$, $\beta_g = \beta_{ph} = 0.995$, $b = 0.25$, and $\Omega = 0.2$.

electric field and perturbed electrons behind the beam are ascended in the presence of inhomogeneity. Furthermore, when the density of the background electrons increases (decreases), the amplitude of the plasma wakefield and perturbed electrons behind the beam increase (decrease). Moreover, it must be noted here that the inhomogeneity and external magnetic field can affect the transformer ratio ($R$), which is related to the maximum energy that can be given to a particle in the witness bunch. It is known that, with increasing the value of the external magnetic field, the transformer ratio, the amplitude of plasma wakefield, and the amplitude of perturbed electron density behind the beam decrease. It is shown in Fig. 1 that, in the case of a constant background electron density, the amplitudes of both the plasma wakefield and the perturbed electron density are constant. However, with increasing (decreasing) inhomogeneity, the amplitudes of plasma wakefield and the perturbed electron density increase (decrease). For a better understanding, the generated plasma wakefields in homogeneous and inhomogeneous plasmas are compared in Fig. 3. Here, we plotted the variation of plasma wakefields in terms of $\xi$ for some of the $\chi$ in Fig. 4. It is evident that the effect of the variation factor of cosine inhomogeneity plays an important role in the excitation of the plasma wakefield. Figure 4 shows that the excited plasma wakefield has a stronger amplitude for smaller values of $\chi$. As we get closer to $\chi = 1$ (corresponding to the homogeneous medium), the amplitude of the excited plasma wakefield becomes lower. In this process, since the effect of the cosine

FIG. 3. Variations of plasma wakefields on homogeneous and inhomogeneous plasmas with a periodical density profile for normalized parameters values. $n_{pe}^0 = 0.5$, $\xi_p = 3$, $\sigma_x = 1.9$, $\beta_g = \beta_{ph} = 0.995$, $b = 0.25$, and $\Omega = 0.2$.

FIG. 4. Variations of plasma wakefields in terms of $\xi$ in an inhomogeneous plasma with a periodical density profile for some of the $\chi$. The beam parameters are $n_{pe}^0 = 0.5$, $\xi_p = 3$, $\sigma_x = 1.9$, $\beta_g = \beta_{ph} = 0.995$, and $\Omega = 0.2$.

FIG. 5. Variations of plasma wakefields in terms of $\xi$ in an inhomogeneous plasma with a periodical density profile for some of the $\sigma_x$. The beam parameters are $n_{pe}^0 = 0.5$, $\xi_p = 3$, $\chi = 0.25$, $\beta_g = \beta_{ph} = 0.995$, and $\Omega = 0.2$. 
FIG. 6. Background density of electrons, the profile of a relativistic Gaussian electron beam, the longitudinal normalized electric field and the amplitude of normalized perturbed electron density behind the beam in a cold inhomogeneous magnetized plasma with a linear density profile. The beam parameters are $n_b = 0.5$, $\xi_p = 3$, $\sigma_x = 1.9$, $\beta_g = \beta_{ph} = 0.995$, $b = -8$, and $\Omega = 0.2$.

function decreases and its profile tends to a straight line, the fluctuations of plasma wakefield amplitude are reduced and the profile is converted into a profile of a homogeneous plasma. We now study the effect of $\sigma_x$ in the relativistic Gaussian electron beam on the generation of plasma wakefields. For this purpose, it is considered that $\sigma_x$ is within the range of 0.1–2. Two-dimensional variations of plasma wakefield in terms of $\xi$ for some of the $\sigma_x$ are shown in Fig. 5. As can be seen from Fig. 5, the plasma wakefield amplitude follows an upward trend from 0.2 to 1 and then a downward trend. In order to have the maximum value of the generated plasma wakefield, the plasma wavelength must have a definite relationship in terms of the length of the drive bunch: $\lambda_{pe} = \sqrt{2\pi\sigma_x}$. For this expansion of the electron beam profile, the wakefield amplitude is maximum when $\sigma_x = \lambda_{pe}/\pi$ or, in terms of dimensionless values, when $\sigma_x = 1$. This implies a good agreement with the results in Fig. 5.

B. Linear density profile

We now consider a cold inhomogeneous magnetized plasma with a linear density profile. This profile is described in Eq. (15). In this case, using the relativistic Gaussian electron beam of Eq. (6), the background electron density, the profile of the electron beam, plasma wakefield, and perturbed electron density are shown in Fig. 6. The amplitudes of wakefields in homogeneous and linearly inhomogeneous plasmas are compared in Fig. 7. This figure shows that

FIG. 7. Variations of plasma wakefields in homogeneous and inhomogeneous plasmas with a linear density profile for normalized parameters values $n_b = 0.5$, $\xi_p = 3$, $\sigma_x = 1.9$, $\beta_g = \beta_{ph} = 0.995$, $b = -8$, $b = 0$, and $\Omega = 0.2$.

FIG. 8. Variations of plasma wakefields in terms of $\xi$ for some of the $b$ in a linearly inhomogeneous plasma. The beam parameters are $n_b = 0.5$, $\xi_p = 3$, $\sigma_x = 1.9$, $\beta_g = \beta_{ph} = 0.995$, and $\Omega = 0.2$.

FIG. 9. Variations of plasma wakefields in terms of $\xi$ for some of the $\sigma_x$ in an inhomogeneous plasma with a linear density profile. The beam parameters are $n_b = 0.5$, $\xi_p = 3$, $\chi = 0.25$, $\beta_g = \beta_{ph} = 0.995$, and $\Omega = 0.2$.
the amplitude of the plasma wave is larger in an inhomogeneous plasma than in a homogeneous plasma. The variations of the generated plasma wakefield in terms of $\xi$ for some of the $b$ are plotted in Fig. 8. It is clear from this figure that, because the background electron density is continuously ascending, the amplitude of plasma wakefield increases continuously with increasing $b$. The effect of $\sigma_0$ on the plasma wakefield amplitude shows a similar behavior, which was seen in Sec. III A for $\sigma_0 = 1$, where the plasma wakefield amplitude is maximum. Of course, by moving away from this point on both sides, the values of the plasma wakefield amplitude become smaller. Now, we study the effect of $\sigma_0$ on the plasma wakefield amplitude, and for this aim, it is considered that $\sigma_0$ varies from 0.2 to 1.8. The generated plasma wakefield is shown in Fig. 9 in terms of $\chi$ for some of the $\sigma_0$.

IV. CONCLUSION

In summary, in this work, we numerically studied the nonlinear interaction of a relativistic Gaussian electron beam with homogeneous and inhomogeneous cold magnetized plasmas. In this process, the analytical equations were expressed and the numerical results were presented. Then, the excitation of plasma wakefield and the amplitude of perturbed electron density behind the beam were compared in a plasma with periodical and linear density profiles. It was shown that in a plasma with period density, the amplitudes of both plasma wakefield and perturbed electron density behind the beam are periodical. In addition, for a medium with a linear density profile, the amplitudes of plasma wakefield and perturbed electron density vary linearly by the variation of the inhomogeneity factor. Moreover, we studied the effect of the bunch length along its propagation direction and the variation factor of inhomogeneity on the generation of plasma wakefields. It was shown that a relativistic Gaussian electron beam in the inhomogeneous plasma can generate a stronger longitudinal electric field than in a homogeneous plasma and the plasma wakefield amplitude is directly related to the background electron density, which is in good agreement with the theoretical and experimental results. Our results showed that the perturbed electron density increases with the plasma wakefield amplitude. It was also shown that for a certain length of the bunch along its propagation direction, the plasma wakefield is maximum, which corresponds to the equality of the plasma wavelength and the electron beam wavelength.

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