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ABSTRACT
Direct numerical simulation is used to study unconfined coaxial jets under the influence of strong swirl imparted to the outer jet. Spectral proper orthogonal decomposition is employed to elucidate the physically important structures or modes in the flow. The analysis is extended to the transport of passive scalars injected through each jet. A partially penetrated vortex breakdown bubble is formed as a result of the strong swirl. In the region upstream of the central stagnation point, the first two (most energetic) spatial modes of the velocity field at the cross-stream section reveal three pairs of counter-rotating vortical structures, while the succeeding two modes reveal four pairs of such structures. The centers of these vortical structures are found to lie in the inner mixing layer present between the two jets. The corresponding spatial modes of the scalars also exhibit organized lobelike structures in this region. These organized structures are subsequently disrupted in the downstream region. The significance of these pairs of counter-rotating vortical structures is demonstrated by reconstruction of various turbulence statistics, namely, the root mean square (rms) velocities, the rms scalar fluctuations, the covariance between the two scalars, and the radial turbulent fluxes of the scalars. The results show that the first four modes make a greater contribution to these statistics except for the covariance between two scalars, particularly in the inner mixing layer.

I. INTRODUCTION
Swirling motion, if imparted to a jet flow, leads to an adverse pressure gradient in the streamwise direction. This results in more rapid decay of the streamwise velocity and enhanced jet growth, as well as improved mixing.1–3 Owing to these characteristics, swirling jets have been used in many engineering applications, specifically in industrial burners and other combustion devices that demand stabilization of flames.1 The efficiency of these devices can be further increased by imparting a strong swirling motion to the jet. If the intensity of swirl exceeds a certain threshold, it results in flow recirculation, a form of vortex breakdown.1,3 This not only prevents the flame from blowing out but also recirculates the products of combustion, which in turn ignites the incoming fuel/air mixture.

Over the years, numerous studies6–9 have reported different types of vortex breakdowns, including helical, axisymmetric bubble, and conical, among others. Moreover, various techniques of modal decomposition have been employed to better understand the physically important structures or modes in swirling jets. These techniques include proper orthogonal decomposition (POD),10,11 dynamic mode decomposition (DMD),12 and spectral proper orthogonal decomposition (SPOD),13 the last of which, as its name suggests, is based on POD. The method of POD focuses on the coherent structures with maximum turbulent kinetic energy,
whereas DMD separates out those coherent structures with a particular frequency. SPOD, however, attempts to leverage the characteristics of both methods, resulting in a clearer representation of the structures involved. Note that the same terminology SPOD is also used by Towne et al.\textsuperscript{13} for their method. However, the two SPOD methods (one proposed by Sieber et al.\textsuperscript{14} and the other by Towne et al.\textsuperscript{13}) differ in the construction of the correlation matrix. Oberleithner et al.\textsuperscript{15} applied POD to a single swirl jet flow and demonstrated the presence of helical structures surrounding the recirculation zone. This was confirmed by Stöhr et al.,\textsuperscript{16} Markovich et al.,\textsuperscript{17} and Percin et al.\textsuperscript{18} with the help of modal decomposition techniques.

Borceé\textsuperscript{19} proposed an extension to the POD technique to investigate the influence of coherent structures on other simultaneously measured physical quantities in the system, such as pressure and scalar concentration. Antoranz et al.\textsuperscript{20} used the extended POD modes to demonstrate a correlation between velocity and temperature in pipe flow. Stöhr et al.\textsuperscript{16} used this approach to correlate the flow and a combustion flame in swirling flow, while Sieber et al.\textsuperscript{14} also employed it in their SPOD analysis.

In our previous study,\textsuperscript{21} the developments of the mean flow and the turbulence statistics were reported for a configuration of unconfined coaxial jets with a strong swirl imparted to the outer jet (OJ). In the present study, we carry out SPOD analysis (proposed by Sieber et al.\textsuperscript{14} since its usability has been explored for the swirling flows\textsuperscript{21–23}) in two-dimensional (2D) planes to investigate the coherent structures present in the flow for the case of strong swirl. Note that the configuration of swirling coaxial jets has rarely been examined using modal decomposition techniques. The effect of the existing flow field structures on passive scalar transport, which has also been scarcely reported, is addressed in the present study. Separate passive scalars are injected through each jet in order to study the mixing between them. A numerical method is used in this study since it is more efficient than experimental measurements when considering the simultaneous transport of two passive scalars.

In Sec. II, we explain the numerical setup along with the precursor simulation for the inlet nozzle to improve the inlet boundary condition. The development of the mean velocity and that of the scalar fields are illustrated briefly in Sec. III along with the relevant code validation. The application of SPOD is presented in Sec. IV. A brief description of the SPOD methodology in Sec. IV A is followed by an illustration of the first few most energetic modes in the fields at various cross sections in Sec. IV B. The influence of low-order modes on the turbulent quantities is then demonstrated in Sec. IV C by reconstructing those quantities. Our conclusions are presented in Sec. V.

II. NUMERICAL SETUP

A. Governing equations, computational domain, and numerical method

The 3D Navier–Stokes and continuity equations are used for the computation of the transient incompressible flow field. The equations, nondimensionalized using the inner jet (IJ) diameter $D$ and inner jet bulk velocity $U_{ij}$, are as follows:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{1}{Re_D} \frac{\partial^2 U_i}{\partial x_j \partial x_j}, \quad (1a)$$

where $U_i$ is the instantaneous velocity component (divided by $U_{ij}$), $p$ is the static pressure (divided by $\rho U_{ij}^2$), $Re_D = U_{ij}D/\nu$ is the inner jet Reynolds number, and $\nu$ is the kinematic viscosity. The passive scalar fields are obtained by integrating the following conservation equations:

$$\frac{\partial \phi_k}{\partial t} + U_j \frac{\partial \phi_k}{\partial x_j} = \frac{1}{Re_D Sc_k} \frac{\partial^2 \phi_k}{\partial x_j \partial x_j}, \quad (2)$$

where $k = 1$ and 2 for the scalars injected through the inner and outer jets, respectively, and $Sc_k$ are the corresponding Schmidt numbers.

Figure 1 shows a schematic of the computational domain. The domain size $L_x \times L_y \times L_z$ is $20D \times 28D \times 28D$. The inner and outer diameters of the outer jet are 1.2$D$ and 2.6$D$, respectively. A wall of thickness 0.1$D$ is present between the two jet exits. A no-slip boundary condition $U_i(x_{boundary}, t) = 0$ is employed for the wall between the exits of two jet nozzles as well as the wall surrounding the outer nozzle. There have been three boundary conditions used by past researchers for the surrounding of the outer nozzle: (i) the coflow [i.e., at the inlet plane, $U_i(x_{boundary}, t) = $ some small value],\textsuperscript{24} (ii) the open boundary (i.e., Neumann boundary condition),\textsuperscript{7} and (iii) the wall boundary.\textsuperscript{7,17} The coflow is not considered in order to avoid the effects of ambient flow such as Kelvin-Helmholtz instability. As for the open boundary condition, it may lead to practically undesired backflow (since fuel/oxidant in the burners could be thrown out of the main flow). Using the wall boundary condition avoids the possible backflow, and hence, it is used in the present study. The technique used for the inlet boundary condition is explained

![FIG. 1. Schematic of the computational domain and inlet configuration. An annular wall of thickness 0.1$D$ is present between the inner jet (IJ) and the outer jet (OJ).](#)
in Sec. II B. The y- and z-directional boundary conditions are taken as the Neumann condition $\partial \Theta / \partial n = 0$, where $\Theta$ refers to the velocity components and scalars. A convective outflow boundary condition is used for the outlet.

The fractional step method is used for solution of the governing equations. The pressure Poisson equation is solved using the conjugate gradient method. A second-order Runge–Kutta scheme is employed for the time advancement. The grid system is of staggered type, equally spaced, and structured. Spatial discretizations of the convection and viscous terms are performed with a conservative scheme and a central difference scheme, respectively. The discretization in the $x$ direction is of fourth-order accuracy, while those in the $y$ and $z$ directions are of second-order accuracy.

The grid points $N_x \times N_y \times N_z$ are of size $700 \times 980 \times 980$, and the time step is equal to $0.001\,92\,D/\langle U \rangle_{IJ}$. Initially, the domain is filled with stationary ambient fluid. The code is written in Fortran and parallelized using the Message Passing Interface (MPI) library. Direct numerical simulation (DNS) is carried out using Fujitsu PRIMEHPC FX100 supercomputer (Information Technology Center, Nagoya University).

The velocity components $U$, $V$, and $W$ are the velocities in the $x$, $y$, and $z$ directions, respectively. Note that the results of the plots are for a line along the $y$ axis, and hence, the components $V$ and $W$ represent the radial and azimuthal velocities, respectively. Time-averaged quantities are denoted by $\langle \cdot \rangle$.

B. Inlet boundary condition

To obtain realistic mean inflow profiles and statistically correct turbulent fluctuations, precursor simulations are performed for the jet nozzles in advance of the main (direct numerical) simulation. This approach involves mapping of the instantaneous velocity at the plane of the nozzle exit onto the inlet of the main simulation. These precursor simulations are carried out using the open-source software OpenFOAM. Large-eddy simulation is used here owing to limitations on the computational resources.

Figure 2 illustrates the nozzle configuration for the inner and outer jets. The inner-jet pipe of diameter $D$ consists of an upstream annular pipe to generate turbulence in the jet. The swirl in the outer jet is generated using four vanes situated on the periphery of the outer-jet pipe in the outer casing. These vanes are separated by equal distances and are tilted by an angle $\alpha$ (which is the angle between the normal to the pipe and the tangent to the vane curvature at the point of contact). In the present study, $\alpha$ is taken as $90^\circ$ to obtain strong swirling motion. The outer casing has four equally spaced inlets of diameter $D$. The Reynolds number of the flow through the inner jet pipe, $Re_{OJ}$, is 2200. The ratio of the total flow rate through the outer jet to that through the inner jet, $Q_{OJ}/Q_{IJ}$, is 10.64, and the resulting bulk velocity ratio $U_{OJ}/\langle U \rangle_{IJ}$ is 2.00. OpenFOAM simulations are carried out using Fujitsu PRIMEGY CX400/270 supercomputer (Information Technology Center, Nagoya University).

Figure 3 shows the radial distributions of the mean velocity components at the nozzle exits along the positive $y$ axis. The mean streamwise velocity $\langle U \rangle$ profile shows peak values of around $1.5 \langle U \rangle_{IJ}$ and $2.3 \langle U \rangle_{IJ}$ for the inner and outer jets, respectively, whereas the mean azimuthal velocity $\langle W \rangle$ profile shows a peak value as high as $2.7 \langle U \rangle_{IJ}$. The swirl number $Sw = \frac{\int_0^{R_o} \langle U \rangle \langle W \rangle r^2 dr}{R_o \int_0^{R_o} (\langle U \rangle^2 - \langle W \rangle^2/2) r dr}$, (3)

where $R_o$ is the outer radius of the outer jet. The numerator and denominator in the above expression represent the streamwise fluxes of azimuthal and streamwise momentum, respectively. Note that the term $-\langle W \rangle^2/2$ in the denominator is an approximation to the static pressure obtained from the radial component of the

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Nozzle configuration. Dimensions are given in terms of the inner jet diameter $D$. (a) Inner-jet nozzle (representative geometry). (b) Outer-jet nozzle. The vane angle $\alpha$ is taken as $90^\circ$.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Radial distributions of the mean streamwise velocity and mean azimuthal velocity at the nozzle exit ($x = 0.0D$).
momentum equation (see Ref. 29 for the formulation). The swirl number calculated at the nozzle exit is 1.8, which is well above the threshold found by Ben-Yeoshua for the occurrence of vortex breakdown.

The libraries of instantaneous velocity components at the jet exits are generated by taking the same time step as that for the main simulation. However, taking account of memory limitation and computational cost, these libraries consist of data corresponding to the total time 76.8\(D/U_{ij}\) and are used repeatedly. In Fig. 4, the accuracy of the transfer (mapping) of data from precursor simulation to DNS is confirmed by comparing the profiles at the nozzle exit in OpenFOAM simulation and the DNS profiles at the first grid point.

Passive scalars, which are not considered in the precursor simulations, are injected with top-hat profiles at the jet exits. The scalar \(\phi_1\) is set to 1 at the inner-jet exit and 0 at the outer-jet exit, while the scalar \(\phi_2\) is set to 1 at the outer-jet exit and 0 at the inner-jet exit. Schmidt numbers for both scalars are set to 1. This also indicates that, in fact, \(\phi_1\) and \(\phi_2\) represent the mass fractions of inner-jet and outer-jet fluids, respectively.

III. DIRECT NUMERICAL SIMULATION

The results for the mean and turbulent quantities are presented in this section to demonstrate the validity of the code and the development of the mean flow. The data used to evaluate these quantities and for the SPOD analysis are extracted over the total time of 96\(D/U_{ij}\) with a span of 0.0096\(D/U_{ij}\), i.e., 10,000 snapshots after the development of flow. However, data over longer time series of 1286\(D/U_{ij}\) with a span of 0.001 92\(D/U_{ij}\) are also used for the purposes of comparison in Sec. III A and could not be used for the SPOD analysis due to its size (snapshots are over 600 000 and maximum grid points at a cross section is around 78 400), making it computationally heavy to handle for the complex calculations of SPOD.

A. Code validation

To validate the numerical results, experiments are conducted with the same nozzle configuration and flow conditions. Particle image velocimetry (PIV) is used for the measurements of the flow field. The test section is illuminated using an Nd:YAG laser (DANTEC RayPower 5000) sheet of a wavelength of 532 nm and a thickness of 1 mm. Seed particles of a mean diameter of 11 \(\mu\)m are dispersed in both jets. A high-speed video camera (Ametek Phantom V210) of 1280 \(\times\) 800 pixel resolution is used for the flow visualization, and the images are captured for 12 s with a sampling frequency of 2000 Hz. The flow field is analyzed using DANTEC DynamicStudio commercial software.

Figure 5 shows the distributions of mean and root mean square (rms) fluctuations of streamwise velocity along the \(y\) axis at various cross sections. For both the mean and rms fluctuations of the streamwise velocity, the profiles of data extracted for the SPOD analysis follow the profiles obtained from longer time series data, indicating the adequacy of the data extracted for SPOD analysis. For the mean streamwise velocity at upstream sections (\(x/D = 0.3\) and \(x/D = 1.0\)), the numerical results show a slight variation as compared with the experimental measurements, and the flow deceleration in the central region (\(|y/D| < 0.5\)) of the experiment is
higher than that of the simulation. This results in early formation of an internal recirculation zone (IRZ) in the experiment. However, the profiles collapse onto each other at the downstream section \(x/D = 3.0\), which is also the region exhibiting the IRZ. The rms fluctuations of the streamwise velocity close to the jet exit are underpredicted by the numerical study as compared with the experimental measurements, but the trends are the same for both. The higher turbulence level in the central region (\(|\phi/D| < 0.3\)) of the section \(x/D = 1.0\) in the experiment may have been caused by early development of the IRZ because the fluctuations intensify as the leading stagnation point approaches. Similar to the mean values, the rms fluctuations of the streamwise velocity also show better agreement between numerical and experimental results at the section \(x/D = 3.0\).

B. Mean velocity and scalar fields

The developments of the mean velocity and scalar fields have been presented in our previous work\textsuperscript{22} and are explained here only briefly. The mean flow development, depicted in Fig. 6, exhibits vortex breakdown in the form of an IRZ. The recirculation occurs in the annular region between the jets as well as prior to the central vortex breakdown bubble (VBB). A similar structure was also found by Santhosh \textit{et al.}\textsuperscript{4} who termed it a partially penetrated VBB. The leading stagnation point on the centerline is situated at \(x = \phi_x = 1.5, \phi_y = 0.3\) of the section \(x/D = 2.5\), and the trailing one is at around \(x/D = 8.0\). Angular momentum is also observed in the recirculation zone.

Figures 7(a) and 7(b) show the development of the mean inner (\(\phi_1\)) and outer (\(\phi_2\)) jet scalars, respectively. A sharp decay in \(\phi_1\) is observed at the leading stagnation point, along with radial spread, which is consistent with the finding of Roback and Johnson.\textsuperscript{30} At the same time, \(\phi_2\) reaches the centerline as far upstream as \(x/D \approx 1.5\), prompted by the IRZ.

IV. APPLICATION OF SPOD

A. Method for structure identification

1. Spectral proper orthogonal decomposition

The SPOD method of Sieber \textit{et al.}\textsuperscript{13} used in the present study resembles the snapshot POD method introduced by Sirovich\textsuperscript{4} except for the prior filtering of the correlation matrix. Consider \(M\) time series data or snapshots collected for \(N\) grid points in a 2D plane with \(M < N\). Here, each snapshot contains the three components (\(U, V, W\)) of the velocity vector \(U\), which can be decomposed into mean (\(U\)) and fluctuating \(U'\) parts. The fluctuating part \(U'\) is arranged in the matrix form as

\[
U' = \begin{bmatrix}
    u'(x_1, t_1) & u'(x_1, t_2) & \cdots & u'(x_1, t_M) \\
    u'(x_N, t_1) & u'(x_N, t_2) & \cdots & u'(x_N, t_M) \\
    u'(x_1, t_1) & u'(x_1, t_2) & \cdots & u'(x_1, t_M) \\
    \vdots & \vdots & \ddots & \vdots \\
    u'(x_N, t_1) & u'(x_N, t_2) & \cdots & u'(x_N, t_M) \\
    w'(x_1, t_1) & w'(x_1, t_2) & \cdots & w'(x_1, t_M) \\
    \vdots & \vdots & \ddots & \vdots \\
    w'(x_N, t_1) & w'(x_N, t_2) & \cdots & w'(x_N, t_M)
\end{bmatrix}
\]

where \(x\) denotes the coordinate and \(t_i\) denotes the \(i\)th snapshot. To find the optimal basis (based on optimizing the mean square of the data variable or turbulent kinetic energy while considering the fluctuating velocity field), the correlation matrix \(R\) is obtained by computing the inner product (denoted by \(\langle , \rangle\)) between every pair of
snapshots (temporal correlation),
\[
R_{ij} = \frac{1}{M} \langle u'(x, t_i), u'(x, t_j) \rangle \tag{5a}
\]
\[
= \frac{1}{M} \sum_{i=1}^{N} [u'(x_i, t_i)u'(x_i, t_j) + u'(x_i, t_i)^T u'(x_i, t_j)] + \sum_{i=1}^{N} a_i(t_j) \Psi_i(x).
\tag{5b}
\]

Thus, the correlation matrix \( R \) is simply equal to \( (u'^T u')/M \) and its size is \( M \times M \). A filtering operation is now performed on the correlation matrix \( R \) to improve the diagonal similarity of \( R \), which allows continuous shifting between the energetically optimal POD and the spectrally clean DMD. A simple Gaussian low-pass filter is given by
\[
S_{ij} = \sum_{k=1}^{M_i} g_k R_{i+k,j+k}, \tag{6}
\]
where \( g_k \) represents the coefficients of the symmetric finite-impulse-response filter of length \( 2M_f + 1 \). Generally, the filter length \( M_f \) can be chosen to correspond to the characteristics time scale, and in the present study, it is taken as an equivalent to the time \( D/U_0 \), leading to \( M_f = 100 \). Here, the Gaussian filter is used for the smooth temporal response taking account of periodicity at the extremes of the time series. It is given by \( g_k = (1/\sqrt{2\pi\sigma}) e^{-(k-1)^2/2\sigma^2} \), with standard deviation \( \sigma = M_f/5 \). The eigenvalues \( \lambda_i \) and eigenvectors \( a_i \) of the filtered correlation matrix \( S \) are then computed by solving the eigenvalue problem. Note that the eigenvalues \( \lambda_i \) represent the modal energies and are arranged in descending order of magnitude \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M \geq 0 \), whereas the eigenvectors \( a_i \) are the temporal mode coefficients. The \( i \)th SPOD spatial modes \( \Phi_i(x) \) are now determined by projecting the snapshots onto the temporal coefficients,
\[
\Phi_i(x) = \frac{1}{M_M} \sum_{j=1}^{M} a_i(t_j) u'(x, t_j). \tag{7}
\]
Finally, the fluctuations in the vector field are reconstructed by
\[
u'(x, t_j) = \sum_{i=1}^{m} a_i(t_j) \Phi_i(x). \tag{8}
\]
where the temporal mode coefficients \( a_i(t_j) \) are the same coefficients that are obtained from the decomposition of the velocity field. It was shown by Borée\textasciitilde{} that these modes give only the part that is correlated with the SPOD velocity modes \( \Phi_i(x) \) while reconstructing the field of scalar fluctuations,
\[
\psi_e(x, t_j) = \sum_{i=1}^{m} a_i(t_j) \Psi_i(x), \tag{10}
\]
whereas the part that is uncorrelated with these modes is given by
\[
\psi_u(x, t_j) = \psi(x, t_j) - \psi_e(x, t_j). \tag{11}
\]
In the present study, \( \psi_u(x, t_j) \) is calculated exactly by prior calculation of \( \psi_e(x, t_j) \) by taking \( m = M \), while the effect of the correlated low-order modes is demonstrated by varying \( m \).

\[
\psi'(x, t_j) = \psi_u(x, t_j) + \sum_{i=1}^{m} a_i(t_j) \Psi_i(x). \tag{12}
\]

3. Data extraction and computation
As mentioned earlier, the number of snapshots taken for SPOD analysis is \( M = 10000 \). The cross sections just upstream and downstream of the central leading stagnation point are focused on in this study. Thus, the data are extracted from the 2D cross sections \( x/D = 0.3, 1.0, 2.5, \) and 3.5. The analysis is carried out using MATLAB.

8. SPOD spatial modes

1. SPOD spatial modes of velocity field

Figure 8 shows the percentage energy content of the SPOD mode of velocity fluctuations \( K = \lambda_i/\sum_{i=1}^{M} \lambda_i \) at the four selected cross sections (i.e., at \( x/D = 0.3, 1.0, 2.5, \) and 3.5). The first four (most energetic) SPOD spatial modes of the velocity field \( \Phi_1, \Phi_2, \Phi_3, \) and \( \Phi_4 \) are then as depicted in Fig. 9, with the contour maps representing the streamwise velocity component and the vectors representing the result of the cross-streamwise velocity components. Red and blue in the contour maps indicate positive and negative values, respectively.

![Figure 8](image-url)
FIG. 9. First four SPOD spatial modes of the velocity field in the $y$-$z$ plane at the four downstream cross sections (a) $x/D = 0.3$, (b) $x/D = 1.0$, (c) $x/D = 2.5$, and (d) $x/D = 3.5$. Contour maps represent the streamwise velocity component. Red and blue colors of contour map represent positive and negative values, respectively, while white color corresponds to the negligible values (this is used for all the figures representing spatial modes hereafter). The contour levels are spanned over $-0.0476:0.0468$ for (a), $-0.0296:0.0293$ for (b), $-0.0193:0.0155$ for (c), and $-0.0155:0.0134$ for (d). At cross section $x/D = 1.0$, the green “+” symbol is used for counterclockwise vortices and the pink “+” symbol is used for clockwise vortices.
At the cross section \( x/D = 0.3 \), the first four modes contribute 6.6% of the total energy (for the first, second, third, and fourth modes, \( K = 2.1\% \), 2.0%, 1.3%, and 1.2%, respectively), and 50% of the total energy is recovered by 141 modes. The first two spatial modes \( \Phi_1 \) and \( \Phi_2 \) resemble each other with a phase shift and thus form the first pair of modes. Three pairs of alternate lobelike structures appear in these two modes at the inner mixing layer (around \( r/D = 0.7 \)). Here, a pair of alternate lobelike structures refers to the alternate regions of positive (red contours) and negative (blue contours) streamwise components of the spatial mode of the velocity field. It can be observed that the positive streamwise component of the spatial mode of the velocity field is associated with the counterclockwise (swirling direction) vectors and the negative one with the clockwise (opposite to swirling direction) vectors. The next two modes \( \Phi_3 \) and \( \Phi_4 \) form the second pair of modes with a phase shift, and these modes contain four pairs of alternate lobelike structures with similar characteristics to those in the preceding modes.

Further downstream at the cross section \( x/D = 1.0 \), the contribution of the first four modes is slightly reduced compared with that at the upstream cross section, to 6.3% of the total energy (for the first, second, third, and fourth modes, \( K = 1.6\% \), 1.6\%, 1.5%, and 1.5%, respectively). It can be seen that these four modes represent approximately equal energy content, unlike that in the upstream cross section, where the first pair of modes represents slightly higher energy content compared with the second pair. Fewer modes (130) are required to recover 50\% of the total energy at this cross section \( x/D = 1.0 \) compared with the upstream cross section. The spatial modes of the velocity field at this downstream cross section \( x/D = 1.0 \) show notable features. Apart from having three pairs of alternate lobelike structures in modes \( \Phi_1 \) and \( \Phi_2 \) (the first pair of modes) and four pairs of alternate lobelike structures in \( \Phi_3 \) and \( \Phi_4 \) (the second pair of modes), three pairs of counter-rotating vortical structures appear in the first pair of modes and four pairs of such structures appear in the second pair of modes. The centers of these vortical structures lie at approximately \( r/D = 0.9 \) (situated at the outer region of the inner mixing layer). Lobelike structures have also been reported in other studies. Störh et al.\(^{19}\) reported a pair of similar lobelike structures for the first two POD modes in their study on cowirling flows. The configuration of coaxial jets with swirl in the outer-jet was studied by Rajamanickam and Basu,\(^{31}\) who also reported the existence of similar three and four pairs of lobelike structures for the first and second pairs of POD modes, respectively, in their pre-vortex-breakdown case (which exhibits a separated recirculation zone surrounding the inner-jet flow).

At the downstream cross section \( x/D = 2.5 \), there is a slightly higher energy contribution \( K \) from the first four modes compared with the upstream cross section \( x/D = 1.0 \) (for the first, second, third, and fourth modes, \( K = 1.8\% \), 1.7\%, 1.6\%, and 1.5\%, respectively). However, the spatial modes of the velocity field suggest that the organized lobelike structures as well as counter-rotating vortical structures are disrupted. Furthermore, modes \( \Phi_1 \) and \( \Phi_2 \) and modes \( \Phi_3 \) and \( \Phi_4 \) are not paired since they exhibit dissimilar structures. At the cross section \( x/D = 3.5 \), the spatial modes of the velocity field are featureless, with no clear evidence of counter-rotating vortical structures.

The flow at the cross sections \( x/D = 1.0 \) and 2.5 is highly turbulent as compared to that at the cross section \( x/D = 0.3 \) (this difference can be observed from the turbulent kinetic energy in Fig. 10). This causes the small scale structures (high-order modes) at cross sections \( x/D = 1.0 \) and 2.5 to possess a larger amount of energy content. Hence, in Fig. 8, the first modes or low-order modes (corresponding to the large scale structures) at the cross sections \( x/D = 1.0 \) and 2.5 have a lower energy content than that at cross section \( x/D = 0.3 \).

The development of the spatial modes (the most energetic four modes) of the velocity field reveals that counter-rotating vortical structures form at the upstream region around the IRZ and eventually fade out in the downstream region. Oberleithner et al., Störh et al.\(^{19}\), Markovich et al.,\(^{18}\) and Percin et al.\(^{16}\) also revealed the presence of vortical structures in the most energetic modes for swirling jet flows and the emergence of the vortical structures was attributed to the helical instability featuring the precession of the vortex core. The effects of these structures on the passive scalar fields and turbulent quantities are discussed in Secs. IV B 2 and IV C, respectively. The cross section \( x/D = 3.5 \) is not considered for further investigation since it shows featureless structures.

### 2. Extended SPOD spatial modes of passive scalar fields

Figures 11 and 12 present contour maps of the first four (most energetic) extended SPOD spatial modes \( \Psi_i(x) \) of the inner- and outer-jet scalar fields, respectively, at cross sections \( x/D = 0.3, 1.0, \) and 2.5. To demonstrate the effect of counter-rotating vortical structures at the cross section \( x/D = 1.0 \), the vector plots from the spatial modes of the velocity field are superimposed on the contour maps of the first and third modes of the scalar fields.

At \( x/D = 0.3 \), the extended spatial modes of both scalar fields exhibit three and four pairs of alternate lobelike structures (although not very clearly) in modes \( \Psi_1 \) and \( \Psi_2 \) (first pair of modes), and \( \Psi_3 \) and \( \Psi_4 \) (second pair of modes), respectively, at the inner...
FIG. 11. First four extended SPOD spatial modes of the inner-jet scalar $\phi_1$ field in the $y$-$z$ plane at the three downstream cross sections (a) $x/D = 0.3$, (b) $x/D = 1.0$, and (c) $x/D = 2.5$. Vectors from the spatial modes of the velocity field are superimposed on the first and third modes at the cross section $x/D = 1.0$ to demonstrate the effect of counter-rotating vortices. The contour levels span $-0.0093$ to $0.0091$ for (a), $-0.0062$ to $0.0078$ for (b), and $-0.0047$ to $0.0026$ for (c).

mixing layer $0.4 \leq r/D \leq 0.7$. Note that the positive mode values of the inner-jet scalar coincide with the negative mode values of the outer-jet scalar in this region, which suggests a negative covariance between the two scalars. This is caused by insufficiency of ambient fluid at this cross section. The spatial modes of the outer-jet scalar show featureless structures at the outer mixing layer ($r/D > 1.0$).

Noteworthy features in the extended spatial modes of both scalar fields are observed at the downstream location $x/D = 1.0$. Three and four pairs of lobelike structures clearly appear in the first and second pairs of spatial modes, respectively, of the two scalars. Moreover, these structures for spatial modes of the inner-jet scalar are confined to the inner mixing layer $r/D \leq 1.2$, while the spatial modes of the outer-jet scalar exhibit distinct patterns in both the inner mixing layer $r/D \leq 1.2$ and outer mixing layer $r/D > 1.2$. It can be observed that positive values of the spatial modes of the inner-jet scalar are predominantly associated with outward (radially) vectors and negative values are associated with inward vectors. In the case of the spatial modes of the outer-jet scalar, the opposite behavior is seen in the inner mixing layer $r/D \leq 1.2$ since positive and negative values of spatial modes are associated with inward and outward vectors, respectively. This reflects the fact that the counter-rotating vortices stimulate outward and inward radial fluxes of the inner- and outer-jet scalars, respectively, which is essential for mixing between the two scalars. However, the structures in the outer mixing layer $r/D > 1.2$ exhibit the outward radial flux of the outer-jet scalar.
since positive values of spatial modes are accompanied by outward vectors.

At the downstream cross section $x/D = 2.5$, the first four spatial modes of both scalars do not show organized structures like those in the upstream sections. Moreover, these spatial modes are not paired with each other.

C. Reconstruction of turbulent quantities

In this subsection, various turbulent quantities are reconstructed from the SPOD modes with $m = 1, 2, 4, 8, 50,$ and 200 at the cross section $x/D = 1.0$ to demonstrate the effect of the organized counter-rotating vortical structures present in the region. These reconstructed turbulent quantities are compared with the quantities evaluated from the original instantaneous data (i.e., a total of 10 000 snapshots).

1. rms velocity fluctuations

Figure 13 presents the radial distributions of the streamwise ($U_{\text{rms}}$), azimuthal ($W_{\text{rms}}$), and radial ($V_{\text{rms}}$) components of the rms velocity reconstructed using $m = 1, 2, 4, 8, 50,$ and 200 and compares them with the respective quantities determined using the original instantaneous data. For $U_{\text{rms}}$, the contribution of the first four modes ($m = 4$) is significantly higher for the upper inner peak at $y'/D \approx 0.9$ (situated in the outer region of the inner mixing layer).
however, it is lower for the lower inner peak at $y/D \approx -0.9$ (also situated in the outer region of the inner mixing layer). Note that the locations of the centers of the counter-rotating vortices also lie at $r/D \approx 0.9$. Thus, the greater contributions of the first four modes can be attributed to the presence of counter-rotating vortical structures. This inequality in contribution between the upper and lower regions is caused by the asymmetrical nature (in the axial direction) of the spatial modes (see Fig. 9). The contribution of low-order modes ($m \leq 8$) is marginal for the outermost peaks at $y/D \approx \pm 1.4$ (situated in the outer mixing layer), which manifests the dominance of the high-order structures (or small-scale structures) in the outer mixing layer region. The modes $m = 200$ are observed to be insufficient to completely recover the profile obtained from data in both the inner and outer mixing layers. Similar trends are also observed in the profiles of $W_{\text{rms}}$ and $V_{\text{rms}}$. Although the contribution from the low-order modes ($m \leq 8$) for the outermost peaks at $y/D \approx \pm 1.4$ is observed to be considerably greater in the cases of $W_{\text{rms}}$ and $V_{\text{rms}}$ compared with $U_{\text{rms}}$, it is still lower compared with the respective inner peaks at $y/D \approx \pm 0.9$ in the cases of both $W_{\text{rms}}$ and $V_{\text{rms}}$. The above discussion demonstrates that the counter-rotating vortices are essentially contributing to the rms velocity peaks in the inner mixing layer ($y/D \approx \pm 0.9$), whereas the high-order structures are the sources of the rms velocity peaks in the outer mixing layer ($y/D \approx \pm 1.4$).

2. rms scalar fluctuations, covariance between two scalars, and radial turbulent flux of scalars

Figure 14 shows the radial distributions of the rms fluctuations of the inner-jet scalar ($\phi_{1,\text{rms}}$) and outer-jet scalar ($\phi_{2,\text{rms}}$) and the covariance between the two scalars ($\langle \phi'_1 \phi'_2 \rangle$) reconstructed using $m = 1, 2, 4, 8, 50, \text{ and } 200$, and compares them with the respective
FIG. 14. Reconstruction of rms fluctuations and covariance between two scalar fluctuations using the first $m$ modes at the cross section $x/D = 1.0$: (a) rms fluctuations of inner-jet scalar; (b) rms fluctuations of outer-jet scalar; (c) covariance between two scalar fluctuations.

quantities evaluated using the original instantaneous data. The first four modes ($m = 4$) contribute significantly to the inner mixing layer peak ($y/D \approx \pm 0.7$) of both $\phi_{1,\text{rms}}$ and $\phi_{2,\text{rms}}$. However, the contribution is less in the lower region ($y/D < 0$) than in the upper region ($y/D > 0$), which is also observed in the case of $U_{\text{rms}}$. A second radial peak is observed for $\phi_{2,\text{rms}}$ in the outer mixing layer region ($y/D \approx \pm 1.5$), and there is also a significant contribution from the first four modes. However, this contribution is lower than that at the inner mixing layer peak. Despite the significant contribution from the first few modes ($m \leq 8$) in both $\phi_{1,\text{rms}}$ and $\phi_{2,\text{rms}}$, the high-order modes (which correspond to the high-order structures) are required to completely recover the profile obtained from the data. As stated earlier in Sec. IV B 2, a negative $\langle \phi_1' \phi_2' \rangle$ is observed at the cross section $x/D = 1.0$, owing to insufficiency of the ambient fluid. In contrast to the rms velocity fluctuations and the rms fluctuations of the scalars, the only radial peak of $\langle \phi_1' \phi_2' \rangle$ (present in the inner mixing layer $y/D \approx \pm 0.7$) has a smaller contribution from the first four modes, and thus, high-order modes are required to recover $\langle \phi_1' \phi_2' \rangle$ obtained from the data.

Figure 15 presents the radial profiles of the radial turbulent fluxes of the inner-jet scalar $\langle v' \phi_1' \rangle$ and outer-jet scalar $\langle v' \phi_2' \rangle$ reconstructed using selected modes and compares them with the respective quantities determined using the original instantaneous data. Note that the radial flux here refers to the flux in the lateral (i.e., $y$) direction. An outward radial flux of the inner-jet scalar is observed in the inner mixing layer ($r/D \leq 1.2$), whereas an inward radial flux of the outer-jet scalar is observed in the same region. As mentioned earlier in Sec. IV B 2, the counter-rotating vortical structures exhibit radial outward and inward fluxes of the inner- and outer-jet scalars, respectively, in the inner mixing layer, and thus,
the contribution from the first four modes is higher for \(\langle v' \phi_1' \rangle\) and \(\langle v' \phi_2' \rangle\) in the upper region \((0 < y/D \leq 1.2)\). However, this contribution is less in the lower region \((0 > y/D \geq -1.2)\), owing to the asymmetry of the spatial modes around the jet axis (see Figs. 11 and 12). Although the first four spatial modes of the outer-jet scalar exhibit an outward radial flux of the outer-jet scalar in the outer mixing layer \((r/D > 1.2)\), its contribution to the total \(\langle v' \phi_1' \rangle\) is negligible. Therefore, high-order modes play a vital role in the flux \(\langle v' \phi_2' \rangle\).

V. CONCLUSIONS

A direct numerical simulation was performed to investigate unconfined coaxial jets under the influence of strong swirl imparted to the outer jet, and spectral proper orthogonal decomposition (SPOD) was employed to elucidate the physically important structures or modes in the flow. The analysis was extended to the transport of two passive scalars that were simultaneously injected through each jet. A partially penetrated vortex breakdown bubble was formed as a result of the strong swirl.

At the cross-stream section present in the region upstream of the central stagnation point \((x/D = 1.0)\), the first two (most energetic) spatial modes of the velocity field revealed three pairs of counter-rotating vortical structures, while the two succeeding modes revealed four pairs of such structures. The centers of these vortical structures were found to lie in the inner mixing layer present between the two jets. Spatial modes (extended) of scalars also exhibited three and four pairs of alternate positive–negative organized lobelike structures in this region for the first two and succeeding two modes, respectively. The vortical structures were observed to result in a radial outward flux of the inner-jet scalar and a radial inward flux of the outer-jet scalar in the inner mixing layer, which is crucial for mixing between the two jets. However, distinct structures appeared in the spatial modes of the outer-jet scalars in the outer mixing layer. These organized structures were subsequently disrupted in the downstream region.

The significance of these pairs of counter-rotating vortical structures was demonstrated by reconstructing various turbulence statistics, namely, the rms velocities, the rms scalar fluctuations, the covariance between the two scalars, and the radial turbulent fluxes of the scalars. The results showed that the first four modes make a greater contribution to these statistics except for the covariance between the two scalars, particularly in the inner mixing layer. However, this contribution was asymmetric about the axis.

The results of this work can be useful for designing practical combustion devices, which often employ swirling flows. As the combustion devices need an efficient mixing of fuel and oxidant, the pairs of counter-rotating vortical structures can be amplified by using an active forcing of flow (such as acoustic forcing) at the inlet plane with the frequency same as the dominant frequency of the temporal mode coefficient \(a_i\) of the modes since these vortical structures are observed to be enhancing the mixing between two jet fluids.

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