Three-dimensional localization of a rotating magnetic dipole from the Fourier integrals of its magnetic flux density with acceleration data

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ABSTRACT

Indoor localization of objects when these bodies are affected by occlusion remains challenging. This paper presents a novel method to estimate the position of a magnetic marker that can be attached to objects such as a robot or a moving vehicle for indoor localization. We derive a formula to compute the azimuth and zenith angles of and the distance to a rotating magnet from the Fourier components of the magnetic flux density at the marker’s rotational frequency. The proposed method does not require temporal changes in the direction of the magnetic moment. As a result, the method does not require communication between the sensor and the marker. Also, using a Fourier component enhances the robustness of the method to environmental noise. The method is verified experimentally and shows an average error of 49.9 mm for estimation of the three-dimensional position of the marker in a 4000 mm by 2000 mm domain where \( z = 350 \text{ mm} \). In addition, we verified that the marker position can be estimated even if the sensor is tilted with respect to the horizontal plane using the sensor’s acceleration data.

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I. INTRODUCTION

Indoor localization is important for various applications such as navigation. However, the fact that the global positioning system couldn’t be used indoors presented a problem. Various methods have been proposed to estimate the indoor position using RFID tags, geomagnetism, and so on. While methods based on simultaneous localization and mapping were proposed, they required map reconstruction when the environment is changed. Also, electromagnetic-wave-based systems have been proposed by Koo and Feng in which the received signal strength of electromagnetic waves was measured. However, the electromagnetic-wave-based method is limited in terms of accuracy because of multi-path effects. When considering the localization of vehicles moving inside a factory or warehouse for preventing human contact accidents, in which there are both people and metal shelves, localization with an accuracy of several tens of millimeters within a range of several square meters would be required.

When compared with these methods, a low-frequency, magnetic-field-based method offers the advantage that it is less strongly affected by both nonferromagnetic objects and human bodies. The conventional method used for localization of a magnetic dipole involves the use of an array of magnetic sensors and dipole localization using the nonlinear least squares fitting method. Paperno et al. generated a rotating magnetic field by inducing quadrature signals in two orthogonal coils. They then estimated the three-dimensional position of a pickup coil by detecting the times at which the norm of the magnetic field vector reaches a maximum and a minimum at the sensor position. Watanabe et al. proposed a method in which a spinning magnet and a smartphone are used as a magnetic marker and a three-axis magnetic field sensor, respectively. The sensor’s azimuth angle is determined by the direction of the rotating magnet when the measured magnetic field strength is at its maximum. The elevation angle and the distance between the magnet and the smartphone are also computed from the magnetic field components. However,
in all these methods, the temporal changes in the direction of the magnetic moment, which is rotated either electrically or mechanically, must be detected. In other words, the phase of the rotating magnetic moment must be known to determine the position of the sensor relative to the marker. This requires communication between the marker and the sensor. Additionally, detection of the maximum magnetic field strength is strongly affected by the signal-to-noise ratio.

In this study, we propose a method to estimate the position of the rotating marker relative to the sensor without knowledge of the temporal changes in the direction of the magnetic moment, which thus eliminates the necessity for communication between sensor and marker. In our method, the azimuth and zenith angles and the distance to the marker are expressed explicitly in terms of the Fourier components of the magnetic flux density at the rotating marker’s frequency. Therefore, by quadrature detection of the measured magnetic flux density, the marker position can be estimated without acquisition of the phase of the rotating magnetic moment. In our previous work, the azimuth angle was restricted to the range from 0 to π; this restriction allowed the sensor to be set on a wall in a room. In this paper, we propose a method that allows this restriction to be removed.

II. METHOD

A. Problem setting

A sensor unit composed of three-axis magnetic sensors and three-axis accelerometers is placed at the origin. The global coordinate system is defined as follows: the z-axis is oriented toward the north using the three-axis magnetic sensors. As shown in Fig. 1, a magnetic marker exists vertically upward direction using three-axis accelerometers and the x-axis is oriented toward the sensor. In our method, the azimuth and zenith angles and the distance to the marker are expressed explicitly in terms of the Fourier components of the magnetic flux density at the rotating marker’s frequency. Therefore, by quadrature detection of the measured magnetic flux density, the marker position can be estimated without acquisition of the phase of the rotating magnetic moment. In our previous work, the azimuth angle was restricted to the range from 0 to π; this restriction allowed the sensor to be set on a wall in a room. In this paper, we propose a method that allows this restriction to be removed.

![FIG. 1. Problem setting.](image)

Under these conditions, the magnetic flux density measured at the origin is expressed as:

$$B(r, t) = \frac{\mu_0}{4\pi} \left( \frac{3(p(t) \cdot r)}{|r|^3} \cdot p(t) \right)$$

where \(\mu_0\) is the free space permeability. We aim to identify the azimuth angle \(\phi\), the zenith angle \(\psi\), and the distance \(r\) to the marker when viewed from the sensor using the magnetic flux density given in Eq. (3). Importantly, we do not know the initial phase, and thus the phase, \(\omega t + \theta_0\), at time \(t\). \(\omega\) and \(p\) are assumed to be known. In practical situations, \(\omega\) can be estimated by fast Fourier transform (FFT) using the measured time series of the magnetic flux density. First, for simplicity, we explain our method for the case where the sensor coordinate system (XYZ-axes) coincides with the global coordinate system (xyz-axes). The method that is applicable when these systems are different is presented in section IIC.

B. Proposed method

We derive a formula to estimate \(\phi\), \(\psi\), and \(r\) in terms of the Fourier components of the magnetic flux density at the rotational frequency \(\omega\). Let us consider \(B_x + iB_y\). From Eq. (3), we obtain

$$B_x + iB_y = \frac{\mu_0}{4\pi} \left( \frac{3(\omega p_x \cdot \mathbf{r}) (x - iy) + (\omega p_x - i\omega p_y)(x + iy)}{r^3} \right) e^{i\theta_0}$$

Substituting Eq. (1) and (2) to Eq. (4), we find

$$B_x + iB_y = \frac{\mu_0}{4\pi} \left( \frac{3 \sin^2 \frac{\psi}{2} e^{-i(\omega t + \theta_0)} + 3 \sin^2 \frac{\psi}{2} e^{i(\omega t + \theta_0)} - 2 \sin \frac{\psi}{2}}{r^3} \right) e^{i\theta_0}$$

Therefore, by quadrature detection of \(B_x + iB_y\), we obtain the Fourier components of the angular frequency \(\pm \omega\) as

$$\int_0^{NT} (B_x + iB_y) e^{-i\omega t} dt = \frac{\mu_0}{4\pi} \left( \frac{3 \sin^2 \frac{\psi}{2}}{r^3} \right) e^{i\theta_0}$$

and

$$\int_0^{NT} (B_x + iB_y) e^{i\omega t} dt = \frac{\mu_0}{4\pi} \left( \frac{3 \sin^2 \frac{\psi}{2}}{r^3} \right) e^{-i\theta_0}$$

where \(T = \frac{2\pi}{\omega}\) and \(N\) is an integer. Also, \(B_z\) is expressed as

$$B_z = \frac{\mu_0}{4\pi} \left( \frac{3 \sin \psi \cos \frac{\psi}{2}}{r^3} \right) \left( e^{-i(\omega t + \theta_0)} + e^{i(\omega t + \theta_0)} \right)$$

and thus we have

$$\int_0^{NT} B_z e^{i\omega t} dt = \frac{\mu_0}{4\pi} \left( \frac{3 \sin \psi \cos \frac{\psi}{2}}{r^3} \right) e^{i\theta_0}$$

From Eqs. (7) and (9), we find
\[
\psi = \arctan \left[ \frac{\int_0^\alpha (B_x + iB_y) e^{i\omega t} dt}{\int_0^\alpha B_z e^{i\omega t} dt} \right],
\]
(10)

\[
\phi = \arg \left[ \frac{\int_0^\alpha (B_x + iB_y) e^{i\omega t} dt}{\int_0^\alpha B_z e^{i\omega t} dt} \right],
\]
(11)

which are used to determine the zenith and azimuth angles, respectively. The distance between the marker and the sensor can be estimated as follows. From Eqs. (6) and (7), we have

\[
\sqrt{\left(\int_0^\alpha (B_x + iB_y) e^{i\omega t} dt\right)^2 + \left(\int_0^\alpha (B_x + iB_y) e^{i\omega t} dt\right)^2} = \frac{p\theta_0 NT}{4\pi r^3} \left(\frac{3\sin^2 \psi}{2} + \frac{3\sin^2 \psi - 2}{2}\right).
\]
(12)

Because Eq. (12) is not equal to zero for an arbitrary value of \(\psi\), the following holds.

\[
r = \sqrt{\frac{p\theta_0 NT}{8\pi} \left(\frac{3\sin^2 \psi}{2} + \frac{3\sin^2 \psi - 2}{2}\right)^2 + \left[\int_0^\alpha (B_x + iB_y) e^{i\omega t} dt\right]^2 + \left[\int_0^\alpha (B_x + iB_y) e^{i\omega t} dt\right]^2}.
\]
(13)

Therefore, if the strength of the moment, \(p\), is known \textit{a priori}, we can then estimate the distance \(r\) by inserting the measured \(\psi\) value into Eq. (13).

These equations hold irrespective of the value of \(\theta_0\), illustrating that the position of the magnetic marker relative to the sensor can be determined from Eqs. (10), (11), and (13) using the Fourier components of \(B_x + iB_y\) and \(B_z\) obtained by quadrature detection of the measured magnetic flux density without knowing the initial phase, and thus the phase at any arbitrary time, of the rotating magnetic moment of the marker.

### C. Coordinate transformation

In this section, we consider the case where the sensor coordinate system does not coincide with the global coordinate system. Let \(\alpha\), \(\beta\) and \(y\) be the angles that the XYZ-axes make with the xyz-axes, respectively.\(^{16}\) Let \(g = (g_x, g_y, g_z)\) be the measured acceleration in the XYZ-coordinate system, where \(\sqrt{g_x^2 + g_y^2 + g_z^2} = 1\), and \(g_0\) is the gravitational acceleration. Let \(B = (B_x, B_y, B_z)\) be the measured geomagnetic flux density, where \(\sqrt{B_x^2 + B_y^2 + B_z^2} = 1\), and \(B_0\) is its strength. Because the gravitational acceleration vector in the xyz-axes is \((0, 0, -g_z)^T\), \(\alpha\) and \(\beta\) are calculated from \((g_x, g_y, g_z)\) using

\[
\sin \alpha = -\frac{g_y}{\sqrt{g_x^2 + g_z^2}}, \quad \cos \alpha = -\frac{g_z}{\sqrt{g_x^2 + g_z^2}},
\]
\[
\sin \beta = g_x, \quad \cos \beta = \sqrt{g_x^2 + g_z^2}.
\]
(14)

Also, because the geomagnetism vector in the xyz-axes is \((B_0, 0, 0)^T\), \(y\) is calculated using \(B_x, B_y, B_z, \alpha \) and \(\beta\) as

\[
\sin y = B_z \sin \alpha - B_y \cos \alpha, \quad \cos y = \frac{B_x}{\cos \beta}.
\]
(15)

Using the \(\alpha, \beta, \) and \(y\) values obtained, the measured magnetic flux density, \(B = (B_x, B_y, B_z)^T\), generated by the rotating magnetic dipole can be transformed into \(B = (B_x, B_y, B_z)^T = R \tilde{B}\), where

\[
R = \begin{pmatrix}
\cos y & \sin y & 0 \\
-\sin y & \cos y & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
0 & \cos \alpha & \sin \alpha \\
-\cos \alpha & 0 & \sin \alpha \\
0 & 0 & 1
\end{pmatrix}.
\]
(16)

Therefore, even when the sensor coordinate system is inclined relative to the global coordinate system, the outputs of the three-axis accelerometers and three-axis magnetic sensors can be used to obtain \(B\) when \(\psi, \phi, \) and \(r\) are estimated from Eqs. (10), (11), and (13), respectively.

### III. EXPERIMENT

#### A. Experimental setup

Figure 2 shows a magnetic marker composed of magnets and a motor (KM-1U, Keigan Inc.). Four pipe-shaped neodymium magnets with length, inner- and outer-diameter of 27 mm, 13 mm and 23 mm, respectively, were connected in series. The magnetic flux density at the magnet’s edge was 50.5 mT. The rotational speed was fixed at 240 rpm (4 Hz).

Two experiments were performed. In experiment 1, the sensor coordinate system coincided with the global coordinate system, but in experiment 2, the systems did not coincide. In experiment 2, the measured magnetic flux density was transformed using the transformation matrix in Eq. (16) before localization. In both experiments, the marker was located at one of the positions marked by asterisks in Figs. 4 and 5 where \(z = 350\) mm.

We used three magnetic impedance (MI) sensors (MI-CB-1DH, Aichi Micro Intelligent Corp.) for a sensor unit as shown in Fig. 3. These sensors were then connected to an oscillator (MI-CB-1DH-OSC, Aichi Micro Intelligent Corp.) to enable synchronous measurement. The sensor outputs were measured using an analog-to-digital converter (NI9215, National Instruments) with a sampling rate of 2 kHz. The FFT of the measured data for 1 s was then computed. The detected peak frequency was 4 Hz \((T = 0.25\ s)\). Using \(NT = 1\), Eqs. (6), (7) and (9) were computed. The zenith and azimuth
angles were estimated from Eqs. (10) and (11), respectively. The average and standard deviation of the errors were evaluated for ten trials.

The magnetic moment, \( p \), was determined as follows. We measured the magnetic flux density at sample points \( r^i = (x^i, y^i, z^i) \), where \( x^i = -1000, -500, \ldots, 1000 \) mm, \( y^i = 1000, 1500, \ldots, 3000 \) mm, and \( z^i = 350 \) mm, and we then solved the linear least-squares problem for \( p \) using Eq. (12).

When the sensor was located close to the marker and the amplitude of the sensor signal exceeded the maximum or minimum output value of the MI sensor, the signal amplitude was estimated as follows. A zero crossover point for the signal was found and was set to be at 0 s. Then, by denoting the signals at time \( t_i \) by \( d_i \) (i = 1, \ldots, \( M \)), which are not saturated around the zero crossover point, the signal amplitude \( A \) and the constant, \( c \), are estimated by solving the linear least-squares problem \( \sum_{i=1}^{M} |d_i - (A \sin(\omega t_i) + c)|^2 \rightarrow \text{min} \).

To measure the gravitational acceleration and the geomagnetism, we used a tri-axial accelerometer and a geomagnetic sensor contained within an Android smartphone (SO-04J, Sony Mobile Communications Inc.). The degree of tilt was measured via the sor contained within an Android smartphone (SO-04J, Sony Mobile). The magnetic moment, \( \psi \), was determined as follows. We measured the magnetic flux density at sample points \( r^i = (x^i, y^i, z^i) \), where \( x^i = -1000, -500, \ldots, 1000 \) mm, \( y^i = 1000, 1500, \ldots, 3000 \) mm, and \( z^i = 350 \) mm, and we then solved the linear least-squares problem for \( p \) using Eq. (12).

For validation of the results, the magnetic marker position was also measured using an optical tracking system (Flex 3, Optitrack). The corresponding results are shown as 'reference' in Figs. 4 and 5.

### B. Results and discussion

Figure 4 and 5 show the two-dimensional estimated position and the reference position in experiment 1 and 2, respectively. The error was calculated as the difference between the reference and estimated positions. The averages and the standard deviations of the absolute errors of three-dimensional localization in experiment 1 and 2 were 49.9 ± 32.3 mm and 129.4 ± 48.8 mm, respectively. The amplitude of the sensor signal was saturated at the only eight points close to the sensor. The measured tilted angles were \( \alpha = 29.6^\circ \), \( \beta = -11.6^\circ \), and \( \gamma = 37.5^\circ \) in experiment 2. Table 1 shows the average and the standard deviation of the absolute errors for the \( x \)-, \( y \)-, and \( z \)-coordinates and the three-dimensional positions in experiments 1 and 2.

In Fig. 4, the marker locations were estimated accurately and stably in the 4000 mm by 2000 mm domains without communicating any information with regard to the temporal change in the magnetic moment direction.

In Fig. 5, the two-dimensional errors when \( x > 0 \) were larger than those obtained when \( x < 0 \). In experiment 2, the sensor coordinate was rotated toward the \( z \)-axis of the global coordinate and the marker approached the \( XY \)-plane of the sensor coordinates. As a result, \( |B_z| \) was smaller than \( |B_x| \) and \( |B_y| \), and the estimated \( y \) and \( z \)-coordinates were less accurate than those in experiment 1. However, the \( x \)- and \( y \)-coordinates were estimated well, as shown in Fig. 5, by transforming the measured magnetic field as described in section II C. This would therefore be sufficient for localization of the magnetic marker that moves in the \( xz \)-plane.

In experiments 1 and 2, \((x^*, y^*, z^*) = (0, 0, 350) \) mm was excluded from the estimation points because \( \int_0^t B_z e^{\omega t} dt \), which is the quantity in the denominator in Eq. (10), is equal to zero when \( \psi \) is zero. However, based on the nature of the situation where the amplitude of \( B_z \) is equal to that of \( B_y \) when \( \psi = 0 \), we can detect \( \psi = 0 \) as follows. First, we detect whether \( \int_0^t B_z e^{\omega t} dt \) is sufficiently
small by setting a threshold. Then, if \[\int_{B_{NT}} B_{0} \omega \omega \, dt \] is nearly equal to 1, we estimate that \(\psi = 0\).

**IV. CONCLUSION**

In this paper, we have proposed a method to estimate the position of a magnetic marker with a rotating magnetic moment. We derived a formula to estimate the azimuth and zenith angles and the distance of the marker from the sensor in terms of the Fourier components at the rotating angular frequency of the magnetic flux density, without knowledge of the direction of the magnetic moment. The position of the marker was estimated with a mean absolute error of 49.9 mm within a 4000 mm by 2000 mm domain, where \(z = 350 \text{ mm}\). Even when the sensor coordinate system did not coincide with the global coordinate system, the position of the marker was estimated using the acceleration data of the sensor with a mean absolute error of 129.4 mm. Therefore, the proposed method can be used in various indoor localization applications.

**ACKNOWLEDGMENTS**

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**REFERENCES**


**TABLE I.** Errors in the estimated position.

<table>
<thead>
<tr>
<th>Case</th>
<th>Error of x (mm)</th>
<th>Error of y (mm)</th>
<th>Error of z (mm)</th>
<th>Three-dimensional error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment 1</td>
<td>27.8 ± 22.4</td>
<td>32.9 ± 30.6</td>
<td>12.5 ± 9.2</td>
<td>49.9 ± 32.3</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>37.5 ± 42.2</td>
<td>52.3 ± 60.6</td>
<td>95.6 ± 19.9</td>
<td>129.4 ± 48.8</td>
</tr>
</tbody>
</table>