Dynamic Q-enhancement in aluminum nitride contour-mode resonators

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Changting Xu, Abhay Kochhar, and Gianluca Piazza

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In this letter, we discuss a dynamic quality factor (Q)-enhancement technique for aluminum nitride (AlN) contour-mode resonators. This technique is implemented by applying an external voltage source that has a specific frequency-dependent phase relationship with respect to the driving voltage source. In this way, the effective spring, damping, and mass of the resonator become dependent on the frequency. With proper gain technique is relatively simple to implement and intrinsically immune to self-oscillations.

Over the past decade, aluminum nitride (AlN) contour-mode resonators (CMRs) have received great research interest as a promising technology for monolithic multifrequency oscillator solutions. However, compared to quartz crystal and capacitive resonators, AlN CMRs exhibit an order of magnitude lower Q. For the center frequency considered in this work, damping is primarily caused by large anchor loss. Efforts on maximizing the device Q passively by reducing energy leakage through anchors have focused on using etched slots/notches, \( \lambda/4 \) suspensions, butterfly-shaped plates, and phononic crystal anchors. However, the absolute Q is bound to be around 5000, which is still 100–1000 \times \) smaller than the fundamental limit intrinsic to the AlN material. By contrast, active Q-enhancement methods can easily break this fundamental limit. These methods were referred to as force feedback and were widely used in scanning probe microscopes (SPMs) to enhance their resolution. Traditionally, active Q-enhancement methods take the device’s real-time current/displacement, amplify it, phase shift it, and feed it back to the device input. The potential issue with this method is related to self-oscillation, which can occur in an active-feedback loop, hence requiring careful selection of the operating point. In addition, the effect of active methods on a resonator electromechanical coupling \( k^2 \) has never been explored. In this letter, we will introduce a dynamic excitation method that eliminates the self-oscillation issue and can also be used to artificially modify the \( k^2 \) of a resonator.

Instead of amplifying and phase shifting the resonator’s displacement or output current and feeding it back to its input, the resonator is excited by two correlated sources at its two separate ports, respectively, as shown in Figs. 1(a) and 1(b). From the perspective of port 1 of the resonator, \( V_1 \) is the driving source, while \( V_2 \) is the external source. The ratio of \( V_2/V_1 \) is chosen to control the behavior of the resonator, because it is easy to implement using an amplifier in series with a delay line, so that

\[
\frac{V_2}{V_1} = G_0 \exp(-j\Omega \tau_0) = G_0 \exp \left( j \frac{\Omega}{\omega_0} \theta_0 \right),
\]

where \( \Omega \) is the spectral frequency, \( G_0 \) a constant real number corresponding to voltage gain, \( \tau_0 \) a constant time delay, and \( \theta_0 = -\omega_0 \tau_0 \) the phase shift at the resonant frequency, \( \omega_0 \). Equivalently, \( V_2 \) can be seen as a voltage-controlled voltage source (VCVS). As we will see later, \( V_2 \) is generated from the same source as \( V_1 \). \( V_2 \) alternatively expands and contracts the AlN body of port 2, effectively exerting an external force on port 1. Conceptually, port 1 can be reduced to a
tance to infinity, i.e., the capacitive capacitance, and resonator of this configuration, $x$

changes over frequency, it is possible to reduce both damping factor and stiffness for port 1 resonator. This is equivalent to a combination of a negative resistance and a positive capacitance presented by port 2, $Z_T(\Omega)$, in series with the motional resistance and capacitance in the electrical domain [Fig. 1(d)]. Hence, $Q$ and $k_2^2$ can be boosted by properly selecting the magnitude and phase of the voltage ratio. For port 1 resonator of this configuration, $k_2^2$ is calculated as

$$k_2^2 = \frac{\pi^2 \omega_r^2 - \omega_s^2}{8 \omega_s^2},$$

where $\omega_r$ is the antiresonant frequency. From Eq. (1), it should already be clear that the magnitude and phase of the voltage ratio can be selected to maximize $Q$, $k_2^2$, or the figure-of-merit (FoM, defined as the product of $Q$ and $k_2^2$) of the resonator. In this letter, we focus on the enhancement in $Q$.

To enhance resonator $Q$, $V_2$ should be appropriately chosen to null $v_1$ at the resonant frequency to boost the magnitude of the admittance to infinity, i.e.,

$$\frac{V_2}{V_1} = -\left[\frac{R_m}{Z_0} + 1 \right] + j \frac{4}{\pi} \frac{1}{k_2 Q} \left( \frac{C_1}{C_0} \right)$$

by assuming $Z_1 = Z_2 = Z_0$, where $Z_0$ is the characteristic impedance, $R_m$ the motional resistance, $C_1$ the printed circuit board (PCB) parasitic capacitance, and $C_0$ the shunt static capacitance between the resonator electrodes. If $G_0$ and $\theta_0$ in Eq. (1) are solely determined by Eq. (3), they are called critical gain and phase in this work, respectively, as such $G_0$ is the minimum gain required to boost $Q$ to infinity when $\theta_0$

is equal to the critical phase (which will be explained in the following paragraph). The FoM for ANI CMRs is typically greater than 10; therefore, Eq. (3) reduces to $G_0 \approx (R_m + Z_0)/Z_0$ and $\theta_0 \approx -\pi$. The need for an external signal that is out-of-phase with the drive can be intuitively explained by looking at Fig. 2. First, $F_{\text{ext}}$ needs to have a 90° phase shift with respect to the displacement of port 1 resonator (denoted as $x_1$ in real-time domain and $X_1$ in complex phasor) at resonance in order to reduce the damping force, which makes $F_{\text{ext}}$ in-phase with $F_{\text{drive}}$. Because of the quarter-wavelength propagation delay from the center of port 2 to the right end of port 1 (the vertical dashed line in Fig. 1(a)), $F_{\text{ext}}$ has an average phase shift of 90° with respect to the resonant displacement of port 2 (denoted as $x_2$ in real-time domain and $X_2$ in complex phasor). This means that $V_2$ (which has a 90° phase shift with respect to $X_2$) has to be out-of-phase with $F_{\text{ext}}$. Therefore, to enable the enhancement in $Q$, $V_2$ has to be out-of-phase with $V_1$ (which is in-phase with $F_{\text{drive}}$).

It is interesting to note that, at resonance, if $G_0$ exceeds a critical gain value, $Q$ decreases because $F_{\text{ext}}$ ends up controlling the dynamics of the system and damps its motion, hence reducing the resonator $Q$. Such property prevents the system from self-oscillation—a clear advantage over other active $Q$-enhancement methods. If $G_0$ keeps constant while the excitation frequency increases, $F_{\text{ext}}$ has a positive projection in $x_1$-direction, which effectively reduces the system’s spring constant, increases its motional capacitance, and enhances the resonator’s $k_2^2$, as shown in Fig. 2(b). Similarly, if $\theta_0$ deviates from the critical phase value, the resonant frequency of the system can be tuned. Specifically, if $\theta_0$ is smaller than the critical phase, $F_{\text{ext}}$ has a positive $x$-direction component, which reduces the effective spring constant and thus $\omega_r$, as shown in Fig. 2(c). Otherwise, $\omega_r$ increases. In addition, these three cases reduce the damping by the same amount as $F_{\text{ext}}$ has the same positive $x_1$-direction component, but the one with critical phase has the smallest $V_2$. Therefore, as mentioned above, the critical gain is the minimum required to boost $Q$ to infinity.

The above intuitive analysis can be confirmed by looking at the quantitative expression of $Z_T(\Omega)$. If $Z_1 = Z_2 = Z_0$, then

$$Z_T = \frac{G_{\text{peak}}}{1 - \frac{1}{G_0^2} Z_{RLC}^2 + \frac{1}{G_0^2} Z_0^2},$$

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$$Z_T = \frac{G_{\text{peak}}}{1 - \frac{1}{G_0^2} Z_{RLC}^2 + \frac{1}{G_0^2} Z_0^2}.$$
The sign of the imaginary parts of $Z_r(\Theta)$ under different situations and the corresponding effect on the resonator behaviors.

<table>
<thead>
<tr>
<th>#Case</th>
<th>$\Omega$</th>
<th>$\Delta \theta_0$</th>
<th>$\Delta \Omega$ or $\Delta \theta$</th>
<th>$\text{Imag}(Z_T)$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>$\omega_i$</td>
<td>= 0</td>
<td>= 0</td>
<td>= 0</td>
<td>No series capacitance added $\Rightarrow$ resonant frequency unchanged</td>
</tr>
<tr>
<td>#2</td>
<td>$&gt;\omega_i$</td>
<td>= 0</td>
<td>&gt;0</td>
<td>&lt;0</td>
<td>Series positive capacitance $\Rightarrow$ antiresonant frequency increased* ↑</td>
</tr>
<tr>
<td>#3</td>
<td>$= \omega_i$</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>&gt;0</td>
<td>Series negative capacitance $\Rightarrow$ resonant frequency reduced ↓</td>
</tr>
<tr>
<td>#4</td>
<td>$&lt;\omega_i$</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>&lt;0</td>
<td>Series positive capacitance $\Rightarrow$ resonant frequency increased ↑</td>
</tr>
</tbody>
</table>

*The antiresonant frequency when boost is applied is given by $\omega'_0 = \left[ L \omega(1/C_p + 1/C_m + 1/C_{\text{CM}}) \right]^{-1/2}$, where $C_m$ represents the additional capacitance in the motional branch due to $\text{Imag}(Z_T)$. $\omega'_0$ is larger than the antiresonant frequency without boost, $\omega_0 = \left[ L \omega(1/C_p + 1/C_{\text{CM}}) \right]^{-1/2}$. Please note that this happens only when the control signal is applied at a frequency $\Omega > \omega_i$ and the equivalent imaginary part of the resonator becomes a function of a frequency. Therefore, based on Eq. (2), $k_1^2$ with boost increases as $\omega_i$ stays the same.

where $\theta = \Omega \theta_0/\omega_i$, $Z_{R/L} = R_M + j \Omega L_M + 1/j \Omega C_M$, and $C_p = C_0 + C_1$. Within the proximity of $\theta$, $\theta = -(\pi + \Delta \theta_0) - \Delta \theta$, where $\Delta \theta = \pi (\Omega - \omega_i)/\omega_i = \pi \Delta \Omega/\omega_i < 1$, and $\Delta \theta_0 < 1$ is the offset phase of $-\theta_0$ from $\pi$ at $\omega_i$. Noting that $\Omega C_2 Z_0 \ll 1$, the real and imaginary parts of $Z_T(\Theta)$ can be approximated by

$$\text{Real}(Z_T) \simeq -\left( \frac{G_0}{1 + G_0} + \frac{R_M - 1}{1 + G_0} \right),$$

$$\text{Imag}(Z_T) \simeq -\frac{\Delta \Omega}{1 + G_0} \left[ \frac{2 G_0 \Delta \theta_0}{1 + G_0} \frac{G_0 \Delta \theta_0}{1} - \frac{\pi G_0 \Delta \theta_0}{1} \frac{\Delta \Omega}{1 + G_0} \frac{G_0 \Delta \theta_0}{1} \right] \frac{G_0 \Delta \theta_0}{1}.$$

Since $G_0$ is set to be greater than 1, $\text{Real}(Z_T)$ is negative in Eq. (5a). Moreover, a full cancelation of $R_M$, i.e., $\text{Real}(Z_T) + R_M = 0$, produces exactly $G_0 = (R_M + Z_0)/Z_0$ in addition. The overall resistance seen from Source $V_1$ is $|Z_0 + R_M + \text{Real}(Z_T)| \approx (R_M + 2G_0)/(1 + G_0) > 0$, which explains why self-oscillation never occurs. Moreover, Eq. (5b) indicates that the imaginary parts of $Z_T(\Theta)$ constitute an either positive or negative capacitance depending on the values of $\Omega$ and $\theta_0$. Table I lists case #1–2 and #3–4 to explain Figs. 2(b) and 2(c), respectively.

Table II shows the design parameters of the AIN CMR that was tested in this work. The width of the device sets the resonant frequency to be 54 MHz. The operation principles and the fabrication process of the device are not described here but can be found in Refs. 30 and 31, respectively. For this device, maximum Q enhancement is attained by setting $G_0$ equal to 3.05 and $\theta_0$ equal to $-178^\circ$ when $Z_1 = Z_2 = Z_0 = 50 \, \Omega$.

Figure 3(a) shows the schematic representation of the active enhancement setup (see Fig. S1 in the supplementary material for details on its implementation). $V_2$ and $V_1$ are generated from the same source input at plane A to drive the device under test (DUT). The DUT performance seen from plane B is of interest; however, only plane A is immediately accessible to the vector network analyzer (VNA) for measurement. Therefore, the calibration at plane B to remove the effect of the path A-B should keep valid when sweeping the gain and phase shift, as well as no signal power should be reflected to plane A from plane C. The details for the calibration and measurement setup to guarantee these two conditions are described in the supplementary material.

We are also interested in the DUT performance seen from plane A. The reflection coefficient at plane A, $\Gamma_A$, can be calculated by the signal flow diagram in Fig. 3(b). According to Mason’s equation,

$$\Gamma_A = \frac{b_A}{a_A} = \frac{\text{S}^{(p)}_{11} + \text{S}^{(p)}_{21} \text{S}^{(p)}_{12} \Gamma_B}{1 - \text{S}^{(p)}_{11} \text{S}^{(p)}_{21} \Gamma_B},$$

where $\Gamma_B$ is the reflection coefficient at plane B. Equation (6) can be simplified as $\Gamma_A \approx \text{S}^{(p)}_{21} \text{S}^{(p)}_{12} \Gamma_B$. Evidently, $\Gamma_A$ depends on whether the signal power at plane A is split equally or unequally into planes B and C. However, $\Gamma_B$ stays unchanged as long as the power difference between planes C and plane B is compensated by the amplifier to ensure the same $V_2/V_1$ is applied to the resonator [see Fig. S1(b) in the supplementary material]. To retain similar Qs at planes A and B, unequal power splitting is preferable. In this letter, the directional coupler (MiniCircuits ZX30-12-4-S+) was employed, whose $|S_{11}^p|^2 \approx |S_{22}^p|^2 \approx 0.92$. Because of this selection, an additional gain of 11.1 dB is required to compensate the power unbalance between plane C and plane B.

Table III and Fig. 4 show the theoretical and experimental results on Q-boosting for different voltage gains at planes B and A. At plane B, with a gain of 3.14 (slightly larger than the critical gain of 3.04) and a phase delay of $-186^\circ$, we can boost Q up to $2.25 \times 10^9$ ($\sim 910 \times$), as shown in Fig. 5(a). Furthermore, the f-Q product of this resonator is effectively $1.22 \times 10^7$ Hz, which surpasses the fundamental limit in
the \( f \)-\( Q \) product of AlN, \( 2.50 \times 10^{13} \) Hz. With an additional gain of \( 11.1 \) dB (\( \equiv 3.6 \)), \( Q \) at plane A can be boosted to \( 1.56 \times 10^{6} \) (\( \equiv 630 \times \)). It is worth noting that the external power can be further minimized by using larger devices or materials with higher coupling coefficient, as \( R_M \) is inversely proportional to \( \gamma_0 \) and \( k_t^2 \). Besides, \( Q \) seen at plane B does not monotonically increase with the voltage gain, as expected, meaning that the system is immune to self-oscillations. The same immunity to self-oscillation at plane A is due to the isolation between planes B and C, which effectively breaks the oscillation loop. In addition, \( k_t^2 \) increases up to \( 7.4 \times 4.4 \times \) at planes B and A, respectively. Also as predicted, the deviation of \( \theta_0 \) from the critical phase tunes the resonator center frequency. The cases with a gain of \( \sim 3.1 \) have increasing resonant frequency when \( |\theta_0| \) is set to smaller values. Such property may be utilized to implement frequency-tuning and hence temperature compensation. Finally, the experimental and theoretical curves overlap well (Fig. 4), although there seems to be a large difference between experimental and theoretical \( Q \) values (Table III). The discrepancy between experimental and theoretical results may be attributed to two reasons: (i) the deviation of the effective \( Z_0 \) from \( Z_0 = 50 \) \( \Omega \) caused by parasitics of cables and connectors that are not properly accounted for, and (ii) when the boosted \( Q \) is high (>20 000), the boosted \( Q \) is very sensitive to small errors in gain and phase shift. Note that much higher \( Q_s \) are also predicted theoretically but for different nominal values of voltage gain (\( G_0 = 3.03 \)) and phase offset (\( \theta_0 = -180^\circ \)). To further clarify this discrepancy, we look at the sensitivity of \( Q \) to gain and phase error [see Fig. 5(b)]. The narrow bright region in Fig. 5(b) indicates that attaining extremely high \( Q \) (>1 000 000) requires an accurate gain and phase shift control. It is intuitive that the larger the \( Q \), the higher the sensitivity to any external parameter. However, for lower \( Q \) but still substantially higher than what is commonly achievable in these resonators (>20 000), large errors in gain and phase can be tolerated (approximately \( \pm 8\% \) for the gain and \( \pm 10^\circ \) for the phase). Finally, it is important to point out that \( Q \)-enhancement in this letter refers to the reduction in a 3-dB bandwidth observed at the termination impedance; however, the ratio of the stored to lost energy (the physical \( Q \)) of the overall system is not improved. Fundamentally, the evaluation of 3-dB \( Q \) in this letter

\[ \text{TABLE III. Theoretical and experimental resonator performances at planes A and B as a function of the relationship between } V_1 \text{ and } V_2. \]

<table>
<thead>
<tr>
<th>( G_0 )</th>
<th>( \theta_0 )</th>
<th>( f_s ) (MHz)</th>
<th>( Q )</th>
<th>( k_t^2 ) (%)</th>
<th>( f_s ) (MHz)</th>
<th>( Q )</th>
<th>( k_t^2 ) (%)</th>
<th>( f_s ) (MHz)</th>
<th>( Q )</th>
<th>( k_t^2 ) (%)</th>
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<tr>
<td>w/o boost</td>
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<td>54.3438</td>
<td>2470</td>
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<td>2200</td>
<td>0.23</td>
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<tr>
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<tr>
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<td>1.13</td>
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<td>1782</td>
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</table>

*The data are calculated based on Eq. (6) using the experimental data at plane B. Although the calibration at plane A could be performed and actual experimental response at plane A could be obtained, it would be hard to directly relate it with the expected results at plane B due to the delay and loss of the cables and directional coupler use to connect to the DUT [see Fig. 5(b) in the supplementary material].
encompasses a system formed by the equivalent VCVS, $V_2$, and the passive resonator. This results in a much higher 3-dB $Q$ than the physical $Q$ of the resonator, i.e., when only passive components are included in the $Q$ evaluation.

In this work, we explored a dynamic Q-enhancement method for AlN CMRs that avoid self-oscillation. With this method, the device’s motional parameters are no longer constant, but depend on the excitation frequency. We demonstrated that the method boosted both $Q$ and $\beta_2$ beyond the intrinsic limit set by material properties at the expense of finite external power consumption. It should be noted that such a method is readily applicable to any other two-port resonators regardless of frequency, mode, and transduction mechanism and could have a direct impact in improving the performance of resonator-based systems for various applications such as filtering and gravimetric sensing.

See the supplementary material for the calibration and measurement setup for the direct demonstration of enhancement in $Q$.

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