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Evaluation of Contact Stresses in the Surface of an Elastic-Plastic Plate Penetrated by a Flat-Ended Rigid Cylindrical Punch

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Abstract. The finite element method is applied to the calculation of normal contact stresses arising in an elastic-plastic plate indented by a flat-ended rigid cylindrical punch, with and without regard for contact interaction. The conformity of the distribution of normal contact stresses on the plate surface is demonstrated for both cases; it is shown that maximum stresses arise at the boundary of the punch-plate contact area and that they have close values. The ratio of normal contact stresses at the boundary of the punch-plate contact area to those in its center, with the used boundary conditions, is 1.8 in the calculation both with and without regard for contact interaction. Consequently, a simplified calculation without consideration of contact interaction provides a qualitative and quantitative evaluation of the distribution of normal contact stresses on the plate surface, and it is applicable to the evaluation of contact stresses when a flat-ended rigid cylindrical punch is indented into a plate made of an elastic-plastic material.

INTRODUCTION

One of the main challenges for the mechanics of solids is the development of methods enabling the stress state of various structural components to be evaluated quickly enough and fairly accurately. However, for bodies of a complex shape, it is more often impossible to obtain elementary formulas for the determination of stresses, strains and displacements. Herewith, modern calculations take into account not only the complexity of body geometry and the diversity of effects (force, temperature etc.), but also the specificity of the physical properties of the body materials. In modern structures, side by side with conventional materials (steel, wood, concrete etc.), new materials possessing some specific properties are widely used. Even in case of conventional materials, it appears necessary to take into consideration plastic properties, e. g. in view of a high level of loading or high temperatures. Therefore, approximate methods for solving problems of the deformation of solids have lately been developed particularly intensively, and the advancement of computer engineering has enabled the achievements of the mechanics of solids to be used in scientific and engineering calculations [1]. Particularly, current FEM-based computer systems allow one to solve a wide range of problems [2-4], including problems on the determination of the stress-strain state of three-dimensional bodies and spatial structures under mechanical loading [5-7].

The classical theoretical boundary problem solution indicates that the value of stress on the boundary of the interaction zone tends to infinity [8]. For example, when a flat-ended rigid cylindrical punch is indented into an elastic plate, the distribution of normal contact stresses $\sigma_{zz}$ under the punch can be calculated by the formula [9]

$$\sigma_{zz}(\rho,0) = -\frac{2GD}{\pi(1-\mu)}(r^2 - \rho^2)^{1/2}, 0 \leq \rho < r,$$

where $G = E/2(1+\mu)$ is the shear modulus, $D$ is indentation depth, $\mu$ is Poisson’s ratio, $E$ is elastic modulus, $r$ is cylinder radius, $\rho$ is the distance from the center of the punch-plate interaction area.
The analysis of this dependence shows that contact stresses at the cylinder edges exceed those in the center, and they tend to infinity when \( \rho = r \). Obviously, the practical distribution of contact stresses due to relaxation will differ from the predicted one. Besides, in the case of elastic-plastic loading, material strain under the punch surface must cause additional stress redistribution.

Thus, the aim of this study is to calculate, with the application of the finite element method, normal contact stresses \( \sigma_{zz} \) arising in an elastic-plastic plate indented by a flat-ended rigid cylindrical punch. Note that, on the one hand, this problem can be solved as three-dimensional contact interaction between two bodies; however, this solution requires a more detailed modeling of the contacting surfaces. On the other hand, the problem can be simplified by an assumption that the cylindrical punch is much stronger than the plate material. In this case, to obtain a solution, it would suffice to specify the punch-plate contact area at the stage of geometric model construction. Therefore, the problem stated is solved for an elastic-plastic plate by calculations with and without regard for contact interaction.

**EXPERIMENTAL PROCEDURE**

The geometric interpretation of the problem is shown in Fig. 1. In the calculation without regard for contact interaction, the following boundary conditions were specified: plate length \( a = 70 \text{ mm} \), plate width \( b = 40 \text{ mm} \), plate thickness \( t = 10 \text{ mm} \), the punch-plate interaction area diameter \( d = 8 \text{ mm} \). The elastic-plastic properties of the plate material was specified in accordance with the bilinear isotropic hardening model with the elastic modulus \( E_1 = 210 \text{ GPa} \), Poisson’s ratio \( \eta_1 = 0.27 \), yield stress \( \sigma_y = 168 \text{ MPa} \) and the tangent modulus \( E_t = 10 \text{ GPa} \). The displacement of the lower bearing face of the plate was assumed zero. The indentation depth was assumed 10 \( \mu \text{m} \). In the calculation with the contact interaction taken into account, the following boundary conditions were additionally specified: the elastic modulus and Poisson’s ratio of the punch material, respectively, \( E_2 = 600 \text{ GPa} \) and \( \eta_2 = 0.23 \) and the punch diameter \( d = 8 \text{ mm} \). The contact interaction was taken into account by specifying the punch-plate contact surface. The friction in the contact zone was ignored. The calculation was made in the Salome-Meca software environment with the Code_Aster solver. The calculation resulted in the determination of the distribution of normal contact stresses \( \sigma_{zz} \) on the plate surface.

**RESULTS AND DISCUSSION**

The results of the calculation of contact stresses in an elastic-plastic plate penetrated by a flat-ended rigid cylindrical punch without and with contact interaction are shown in Figs. 2 and 3. The absence of pronounced stress peaks and a specific symmetry point to a high quality of discretization. It is obvious from Figs. 2 and 3 that the normal contact stresses in the center of the punch-plate interaction area are minimum and amount to \( -257 \text{ MPa} \). As the normal contact stresses approach the boundary of the interaction area, they gradually increase, and at a distance of 2.9-3.0 mm from the center of the contact area there is a sharp increase in the normal contact stresses. Maximum normal contact stresses arise at the boundary of the punch-plate interaction area. Note that, for the elastic-plastic plate, in the calculation with and without regard for contact interaction, these stresses have close values amounting to \( -456 \text{ MPa} \).
Thus, when the flat-ended rigid cylindrical punch is indented into the elastic-plastic plate, the normal contact stresses increase as the interaction area boundary is approached, this being in agreement with the theoretical dependence. The maximum normal contact stresses arise on the punch-plate interaction boundary and, with the used boundary conditions, the ratio of normal contact stresses on the boundary of the punch-plate interaction area to those in its center is 1.8 in the calculation both with and without regard for contact interaction. Consequently, the simplified calculation without regard for contact interaction enables the distribution of normal contact stresses on the plate surface to be evaluated qualitatively and quantitatively, and it is applicable to the evaluation of contact stresses arising in an elastic-plastic plate indented by a flat-ended cylindrical punch.

CONCLUSION

Normal contact stresses arising in an elastic-plastic plate indented by a flat-ended rigid cylindrical punch have been calculated with the application of the finite element method, with and without regard for contact interaction. It
has been demonstrated that the distributions of normal contact stresses on the plate surface for the two considered cases conform to each other, that the maximum stresses arise on the boundary of the punch-plate interaction area and that they have close values. The ratio of normal contact stresses on the boundary of the punch-plate interaction area to those in its center, with the used boundary conditions, is 1.8 in the calculation both with and without regard for contact interaction. Consequently, a simplified calculation, ignoring contact interaction, provides a qualitative and quantitative evaluation of the distribution of normal contact stresses on the plate surface, and it is applicable to the evaluation of contact stresses arising in an elastic-plastic plate penetrated by a flat-ended rigid cylindrical punch.

ACKNOWLEDGMENTS

This study was financially supported by the Russian Foundation for Basic Research, grant No. 15-08-06754_a.

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