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Cite as: AIP Conference Proceedings 1910, 030006 (2017); https://doi.org/10.1063/1.5013965
Published Online: 07 December 2017

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A DG Approach to the Numerical Solution of the Stein-Stein Stochastic Volatility Option Pricing Model

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Abstract. Stochastic volatility models enable to capture the real world features of the options better than the classical Black-Scholes treatment. Here we focus on pricing of European-style options under the Stein-Stein stochastic volatility model when the option value depends on the time, on the price of the underlying asset and on the volatility as a function of a mean reverting Ornstein-Uhlenbeck process. A standard mathematical approach to this model leads to the non-stationary second-order degenerate partial differential equation of two spatial variables completed by the system of boundary and terminal conditions. In order to improve the numerical valuation process for a such pricing equation, we propose a numerical technique based on the discontinuous Galerkin method and the Crank-Nicolson scheme. Finally, reference numerical experiments on real market data illustrate comprehensive empirical findings on options with stochastic volatility.

INTRODUCTION

Financial engineering tasks commonly require sufficient knowledge of advanced mathematical methods. Obviously, a method originally developed for one task can be worth to implement for another task. It is also quite common that methods designed for the analysis of problems in, say, physics can be easily adjusted for efficient applications within the issues of financial modelling.

The ability to apply proper methods in the right way is a necessary condition of good performance for any entity active at financial markets. Moreover, quick implementation of bright new approaches and algorithms can bring a given entity competitive advantage against its competitors.

As an example, let us consider options, a specific type of financial derivatives valuation of which often requires knowledge of advanced techniques. The reason is that options give the owner a right (but not the obligation) to make some particular trade with the underlying asset at maturity time. It follows that the value function is not linear and in some regions it is very sensitive to the changes of input factors.

As the general foundations of modern theory of option pricing we generally accept the ideas published by Black and Scholes [1] and Merton [2], i.e., construction of riskless portfolio of the option and its underlying asset (or, potentially, further risk sources). The solution of the problem has, however, been closely related to the solution of the heat equation well known in physics, i.e., partial differential equation (PDE) with proper boundary and terminal (initial) conditions.

Since the system can lead to analytical option pricing formula only under some simplifying assumptions, it is often inevitable to adopt some of the numerical approximation techniques. In this contribution we further extend our previous work on numerical solution of option pricing problem using the discontinuous Galerkin (DG) method [3, 4] as an alternative to the common approaches based on finite elements [5] or the recent wavelet approach from [6] following paper [7].

In particular, our aim is to suggest an alternative to the DG numerical scheme for valuation of European options under Heston stochastic volatility approach presented in [8] – the Stein-Stein model [9]. In principle, the newly proposed scheme is similar to the one presented in [8] and thus it arises from the discretization of the corresponding PDE for simple vanilla options.
We proceed as follows. First, we recall the Stein-Stein model, which relaxes the assumption of constant volatility into stochastic volatility model, and introduce the corresponding initial-boundary value problem of two-factor model. Subsequently, we realize the DG discretization and present two numerical benchmarks.

**STEIN-STEIN STOCHASTIC VOLATILITY MODEL**

The Stein-Stein stochastic volatility model was firstly introduced in the original paper [9] of E.M. Stein and J.C. Stein as a one possible generalization of the Black-Scholes (BS) framework by letting the volatility itself be modeled as an arithmetic Ornstein-Uhlenbeck (OU) process instead of assuming its constant value. We consider here a slight modification, since the asset price process $S$ and the volatility process $Y$ satisfy the following system of stochastic differential equations at time $t$:

\[
\begin{align*}
    dS_t &= \mu S_t dt + \sigma S_t dW_t, \quad S_0 \geq 0, \\
    dY_t &= \alpha (m - Y_t) dt + \beta dZ_t, \quad Y_0 \geq 0,
\end{align*}
\]

where $W(t)$ and $Z(t)$ are two independent Wiener processes, i.e., $dW_t dZ_t = 0$. The system (1)–(2) is calibrated with four fixed constants, namely: $\mu \in \mathbb{R}$, a drift rate of the asset price process; $\alpha > 0$, a rate of mean reversion; $m > 0$, a long term mean of the volatility process; $\beta > 0$ an asymptotic volatility of volatility. For a detailed explanation of these market parameters the reader is referred to the book [10]. It is important to mention that volatility is governed by OU process (2), which is not positive in general. Therefore, absolute value of $\frac{\partial V}{\partial Y}$ has to be taken into account for the description of asset price in (1). More precisely, in the absence of correlation, the model given by (1)-(2) coincides with the original one from [9].

Next, we consider a European option on the financial asset $S$ with maturity $T$ and assume instantaneous risk-free interest rate $r > 0$. Moreover, consider reversal time running $t = T - \tilde{t}$, then we denote by $V(S, Y, t)$ the price of this option at time $t$, which will depend on the underlying asset $S$, the driving process $Y$ and the time to maturity $T$. By using a common approach based on no arbitrage principle, Ito’s stochastic calculus and a construction of a riskless portfolio (see [11]), it is possible to prove that the pricing function $V$ satisfies the following PDE

\[
\frac{\partial V}{\partial t} - \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rS \frac{\partial V}{\partial S} + rV - \frac{1}{2} \beta^2 \frac{\partial^2 V}{\partial Y^2} + \rho \frac{\partial V}{\partial Y} = 0, \quad S > 0, \quad Y \in \mathbb{R}, \quad 0 < t \leq T,
\]

where the term $\Lambda(S, Y, t)$ is usually defined as

\[
\Lambda(S, Y, t) = \rho \frac{\mu - r}{|Y|} + \sqrt{1 - \rho^2} \gamma(S, Y, t),
\]

where $\rho \in [-1, 1]$ stands for the correlation factor and the function $\gamma(S, Y, t)$ is the risk premium factor (i.e., return on the volatility risk), which can be chosen arbitrarily, see [11]. Taking this into account together with the independence of both processes in (1) and (2), i.e., $\rho = 0$, we take $\Lambda(S, Y, t) = 0$ for the rest of the paper, without loss of generality.

Since the pricing PDE (3) is of the first order in time, it is necessary to prescribe the initial condition given by the payoff function of the corresponding European vanilla option at maturity date $T$, i.e.,

\[
V(S(0), Y(0), 0) = V_0(S, Y) := \left\{ \begin{array}{ll}
    V^C_0(S, Y) = \max(S - K, 0), & \text{if } V \text{ is call}, \\
    V^P_0(S, Y) = \max(K - S, 0), & \text{if } V \text{ is put},
\end{array} \right.
\]

where $K$ denotes the strike price. Although, the Cauchy problem (3) with (5) is defined on unbounded spatial domain, the numerical treatment has to be performed on the initial-boundary value one.

**Initial-Boundary Value Problem**

At first, we truncate $S$-domain on rectangle $(0, S_{\text{max}}) \times (-Y_{\text{max}}, Y_{\text{max}})$ with $S_{\text{max}}$ and $Y_{\text{max}}$ large enough and denoting the maximal sufficient values of the asset price and its volatility, respectively. Further, in practice, we encounter the situation $S_{\text{max}} \gg Y_{\text{max}}$, thus one can avoid the degeneracy of a spatial domain in a straight line by a suitable change of variables. Setting $x = [x_1, x_2] = [S/K, Y] \in \Omega = (0, S_{\text{max}}/K) \times (-Y_{\text{max}}, Y_{\text{max}})$, the option price and initial states transform into

\[
\begin{align*}
    u(x, t) &= V(S, Y, t)/K, \quad u^C_0(x) = V^C_0(S, Y)/K = \max(x_1 - 1, 0), \quad u^P_0(x) = V^P_0(S, Y)/K = \max(1 - x_1, 0) \quad (6)
\end{align*}
\]
and the governing equation (3) for the pricing function \( u(x, t) : \Omega \times (0, T) \rightarrow [0, +\infty) \) can be rewritten in the divergence form as
\[
\frac{\partial u}{\partial t} - \text{div} (D(x) \cdot \nabla u) + \nabla \cdot f(x, u) + c(x)u = 0 \quad \text{in} \ \Omega \times (0, T). \tag{7}
\]
In the standard terminology of theory of PDEs the equation (7) is a degenerate second order equation due to the presence of the symmetric positive semi-definite matrix
\[
D(x) = \left( \begin{array}{cc}
d_{11}(x) & d_{12}(x) \\
d_{12}(x) & d_{22}(x)
\end{array} \right) = \frac{1}{2} \begin{pmatrix} x_1^2 & \beta^2 x_2 \\
\beta^2 x_2 & 0
\end{pmatrix}. \tag{8}
\]
The vector \( f(x, u) = (f_1(x, u), f_2(x, u))^T \) represents a physical flux with components
\[
f_1(x, u) = \left( \frac{\partial d_{11}}{\partial x_1}(x) + \frac{\partial d_{12}}{\partial x_2}(x) - rx_1 \right) u = (x_2^2 - r)x_1 u, \tag{9}
\]
\[
f_2(x, u) = \left( \frac{\partial d_{12}}{\partial x_1}(x) + \frac{\partial d_{22}}{\partial x_2}(x) - a(m - x_2) \right) u = a(x_2 - m)u \tag{10}
\]
and the scalar function
\[
c(x) = 2r - a - \frac{\partial^2 d_{11}}{\partial x_1^2}(x) - \frac{\partial^2 d_{12}}{\partial x_1 \partial x_2}(x) - \frac{\partial^2 d_{22}}{\partial x_2^2}(x) = 2r - a - x_2^2 \tag{11}
\]
stands for the variable reaction coefficient balancing the original equation (3) rewritten into the divergence form (7).

In what follows, we discuss the choice of boundary conditions. The setting of these conditions plays an important role to achieve highly accurate solutions. Unfortunately, one can find in the literature many different types of boundary conditions and there is no consensus regarding boundary conditions for PDE stochastic volatility models. In our study we follow the common approach, where the boundary conditions are chosen in accordance with the algebraic sign of so-called Fichera function (for a detailed explanation see [12]) defined on the boundary \( \partial \Omega \), which describes the direction of the convective flow on \( \partial \Omega \). Let \( \partial \Omega \) consist of four disjoint parts
\[
\Gamma_1 = [0, S_{max}] \times [-Y_{max}], \quad \Gamma_2 = [S_{max}] \times (-Y_{max}, Y_{max}), \quad \Gamma_3 = (0, S_{max}) \times \{Y_{max}\}, \quad \Gamma_4 = [0] \times (-Y_{max}, Y_{max}). \tag{12}
\]
Then the corresponding boundary conditions are of a mixed type and read as:

- on \( \Gamma_1 \cup \Gamma_3 \): From the financial standpoint, the relation \( \frac{\partial \tilde{u}}{\partial n} \gg \beta^2 \) is not violated on \( Y = \pm Y_{max} \), thus the convection term dominates diffusion one and it has a velocity pointing outward the domain \( \Omega \). Therefore, it is possible to set an artificial boundary condition of Dirichlet type given by the extrapolated value of the solution \( u \) from \( \Omega \) (i.e., \( u_{ext} \)) to the corresponding parts of the boundary, i.e.,
\[
u(x, t) = u_{ext}(x, t) \quad \text{on} \ \Gamma_1 \cup \Gamma_3, \ t > 0. \tag{13}
\]
- on \( \Gamma_2 \): We prescribe Neumann boundary condition corresponding to slopes of European options as \( S \to +\infty \). It means that the call option has a unit slope with respect to the asset price \( S \) and the put option a zero slope, respectively. Therefore, we set
\[
(D(x) \cdot \nabla u(x, t)) \cdot \vec{n} = \begin{cases} d_{11}(x), & \text{for call,} \\ 0, & \text{for put,} \end{cases} \tag{14}
\]
where \( \vec{n} \) is the outer unit normal.
- on \( \Gamma_4 \): In order to guarantee the well-posedness of the initial-boundary value problem, no boundary condition has to be imposed on \( S = 0 \), because \( \frac{\partial \tilde{u}}{\partial n} = 0 \) on \( \Gamma_4 \), i.e., Fichera function becomes zero, see [11].

Finally, note that PDE (7) is convection-dominated when \( |x_2| \) is large. Therefore, one has to take into account this feature when numerical solution is constructed, e.g., an incorporation of the upwinding technique, see next section.
NUMERICAL SOLUTION OF THE PRICING PDE

We apply the DG framework (see [4] for a complete overview) to the initial-boundary value problem (7)–(11) with (5) and (13)–(14) in order to utilize the potential of this method for solving such option pricing problems with stochastic volatilities. We construct solution $u_h = u_h(t)$ from the finite dimensional space $S_h^p$ consisting from piecewise polynomial, generally discontinuous, functions of the $p$-th order defined on the domain $\Omega$. Using a method of lines, cf. [13], leads to a system of the ordinary differential equations for unknown function $u_h$, i.e.,

$$
\frac{d}{dt} (u_h, v_h) + A_h(u_h, v_h) = l_h(v_h)(t) \quad \forall \, v_h \in S_h^p, \quad \forall \, t \in (0, T),
$$

(15)

where the initial condition $u_h(0)$ is given by (6), $(\cdot, \cdot)$ denotes the inner product in $L^2(\Omega)$, the bilinear form $A_h(\cdot, \cdot)$ stands for the DG semi-discrete variant of degenerate parabolic partial differential operator from (7) and the form $l_h(\cdot)(t)$ arises from boundary conditions, for more details see [14].

Since we want to propose the high-order scheme also with respect to the time coordinate $t$, we realize the discretization in time by the trapezoidal rule giving the second order convergence in time. Actually, it is the average of forward and backward Euler scheme in time, well-known as the Crank-Nicolson method, see [15]. We consider the time discretization in time by the trapezoidal rule giving the second order convergence in time. Actually, it is the average of forward and backward Euler scheme in time, well-known as the Crank-Nicolson method, see [15]. We consider the equidistant time partition $0 = t_0 < t_1 < \ldots < t_s = T$ with the time step $\tau$ and define the DG approximate solution of problem (15) as functions $u_h^m = u_h(t_m)$, $t_m \in [0, T]$, satisfying the following numerical scheme for $m = 0, 1, \ldots, s - 1$,

$$
(u_h^{m+1}, v_h) + \frac{\tau}{2} A_h(u_h^{m+1}, v_h) = (u_h^m, v_h) - \frac{\tau}{2} A_h(u_h^m, v_h) + \frac{\tau}{2} \left( l_h(v_h)(t_m) + l_h(v_h)(t_{m+1}) \right) \quad \forall \, v_h \in S_h^p,
$$

(16)

with the starting data $u_h^0 = u_h(0)$. Note that scheme (16) is the specific case of so-called $\theta$-scheme, compare with [16] or [17] regarding the wavelet approach to option pricing.

Moreover, one can easily identify that the numerical scheme (16) corresponds to the system of linear equations. More precisely, we rewrite the discrete DG solution $u_h^m$ as a linear combination of basis functions, i.e.,

$$
u_h^m(x) = \sum_{k=1}^{\text{DOF}} \xi_k^m \varphi_k(x), \quad x \in \Omega, \quad \text{and} \quad S_h^p = \mathcal{L}(\varphi_1, \ldots, \varphi_{\text{DOF}}),
$$

(17)

where $\text{DOF}$ denotes the number of degrees of freedom and $\mathcal{L}$ the linear span, respectively. Next, we set the vector of real coefficients of basis functions $U_m = (\xi_k^m)_{k=1}^{\text{DOF}} \in \mathbb{R}^{\text{DOF}}$, then, from (16) we receive the sparse matrix equation for an unknown vector $U_m$ related to the DG solution $u_h^m$, i.e.,

$$
(M + \frac{\tau}{2} A) U_{m+1} = \left( M - \frac{\tau}{2} A \right) U_m + \frac{\tau}{2} (F_{m+1} + F_m),
$$

(18)

where the matrix $M$ corresponds to the mass matrix, the matrix $A$ arises from the bilinear form $A_h$ and the vector $F_m$ enforces the fulfillment of boundary conditions at time level $t_m$, respectively.

TWO NUMERICAL BENCHMARKS

We want to price European put and call options under the Stein-Stein model. The computations below are done by software Freefem++ [18], in which the proposed DG approach is implemented. We use piecewise linear, quadratic and cubic approximations on structured and unstructured triangular meshes. These unstructured grids are constructed by adaptively refinement from the structured initial mesh. This procedure allows us to resolve well the obtained numerical solution in the whole domain. During all simulations, we use a constant time step proportional to one trading day, i.e., $\tau = 1/250$, and GMRES as a sparse solver for (18).

Put Option

The first benchmark is chosen according to [11] to provide experimental insight into the mutual comparison of the proposed DG approach with the finite element schemes. To be consistent with the reference experiment we consider European put with strike $K = 20$, one year maturity and the following model parameters:

$$
r = 0.05, \quad \alpha = 1.0, \quad m = 0.2, \quad \beta = \frac{1}{\sqrt{2}}, \quad S_{\text{max}} = 10K, \quad Y_{\text{max}} = 3.0,
$$

(19)
where $S_{\text{max}}$ and $Y_{\text{max}}$ are chosen sufficiently large to be far enough from the zone of practical interest. Then we are able to use boundary conditions (13) and (14) on far-field boundary without a great influence on the numerical solution in the rest of the computational domain. For this reason, we take values of $Y_{\text{max}}$ greater than one even though these values are useless from a financial viewpoint. Also note that the choice of a unit mean reversion rate (i.e., $\alpha = 1.0$) is not quite realistic from a financial point of view, because this value is generally larger in practice. However, smaller values of $\alpha$ could be reasonable, if the underlying asset $S$ corresponds to interest rates, see [11].

In Figure 1, we show the initial state given by the payoff function (5) and Figure 2 shows the resulting DG approximation of a pricing function $V$ at maturity. Note that this final solution is not symmetric in the variable $Y$ as it might seem, because $m \neq 0$. These solutions are obtained with the proposed DG scheme (16) employing linear polynomials. Moreover, during the simulation, the mesh is refined adaptively every trading month, from the initial grid having 800 elements to the final one with approximately 2,700 elements. This refinement is driven by the orientation of convection flux (9)-(10) and it is better observable on plots with isolines of solutions. Finally, one can easily conclude that we are in a good agreement with reference solutions from [11], obtained by the finite element method.

**FIGURE 1.** Piecewise linear approximation of the initial condition: 3D plot (left) and the corresponding isolines (right).

**FIGURE 2.** Piecewise linear approximation of the final state: 3D plot (left) and the corresponding isolines (right).

### Call Option

The second benchmark is performed on reference market data from the original pioneering paper [9]. Our aim is to approximate a call option and investigate the behaviour of option values with respect to different strikes as well as maturities. The remaining model parameters are fixed, i.e.,

$$r = 0.0953, \quad \alpha = 16.0, \quad m = 0.25, \quad \beta = 0.4, \quad S_{\text{max}} = 600, \quad Y_{\text{max}} = 1.0, \quad S_{\text{ref}} = 100, \quad Y_{\text{ref}} = 0.25, \quad (20)$$
where $S_{ref}$ and $Y_{ref}$ determines the coordinates of a reference node. To capture several scenarios we consider seven values of $\mathcal{K} \in \{90, 95, 100, 105, 110, 115, 120\}$ and three maturities with lengths of one month, one quarter and half-year.

For all cases, we compute solutions on one fixed structured grid having 15,360 elements (spacing $480 \times 16$) with piecewise linear, quadratic and cubic approximations. The comparative results are evaluated at given reference node $[S_{ref}, Y_{ref}]$ and recorded in Table 1 along with the values from [9], which can be considered as the reference ones. This table is divided into three panels corresponding to particular maturity. The obtained results are of higher accuracy as the polynomial order increases and give fairly the same values as in [9]. For a complete overview, BS prices are also included in Table 1 as nonstochastic volatility setting where $\alpha = 0$ and $\beta = 0$. Finally, let us state that our observations illustrate empirical findings common for the European option priced under the Stein-Stein stochastic volatility model.

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CONCLUSION AND REMARKS

In this contribution we have extended our previous results on numerical approximation to (vanilla) option value via DG numerical scheme with stochastic volatility. In particular, we have considered the Stein-Stein model as an alternative to Heston stochastic volatility model. The results of experimental studies show quite good accordance with selected benchmarks. Obviously, proposed scheme can be further extended to deal with other options and processes in order to fully utilize the advantages of DG approach; however, in such cases it is not so easy to obtain benchmarks for the comparison.

ACKNOWLEDGMENTS

Both authors were supported through the Czech Science Foundation (GACR) under project 16-09541S. The support is greatly acknowledged. Furthermore, the second author also acknowledges the support provided within SP2017/32, an SGS research project of VSB-TU Ostrava.
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