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Guidance of surface elastic waves along a linear chain of pillars

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(Received 30 September 2016; accepted 6 December 2016; published online 19 December 2016)

The propagation of surface elastic waves, or surface phonons, is considered along a linear and periodic chain of cylindrical pillars sitting on a semi-infinite solid substrate. A variety of guided modes, some of them exhibiting a very low group velocity, are shown to exist at frequencies close to the resonance frequencies of the pillars. Although the pillar diameter is typically smaller than half the relevant wavelength, lateral radiation on the surface is found to be canceled. Surface guidance is explained by the hybridization of the resonating pillars with the continuum of elastic waves of the substrate. © 2016 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). [http://dx.doi.org/10.1063/1.4972552]

I. INTRODUCTION

The problem of guiding elastic waves or acoustic phonons along a surface is of great fundamental and practical interest. Surface acoustic wave (SAW) devices are for instance widely used for analog radiofrequency signal processing in modern telecommunication systems. These devices usually rely on the use of Rayleigh waves or other types of surface guided waves traveling on a homogeneous surface that only provide guidance for straight-crested waves propagating in very large beams. A much tighter control of the propagation of surface waves can be achieved by surface structuration that allows to obtain guidance along a very narrow path on the surface, so that wave confinement is maximized. Various strategies have been proposed over the past decades in order to achieve this goal. Phononic crystals1 can for instance be used to implement waveguiding functions through the introduction of a linear defect in 2D crystals of holes.2–4 In this case, a waveguiding band appears in a frequency range that is contained within a complete band band gap of the phononic crystal. Waveguides relying on a local resonance of a 2D lattice of pillars on a surface have also been demonstrated.5–10 In both types of phononic crystal waveguides, though guidance is obtained inside a defect the width of which is smaller than the wavelength, the full structure still has to cover a larger part of the surface as a minimal number of phononic crystal layers have to be present on both sides of the waveguide to ensure wave confinement.

In order to obtain a minimal width for the waveguide, we choose here to release one direction of periodicity and consider guidance along a narrow linear chain of pillars. The linear chain then forms a 1D surface phononic crystal. The idea of using 1D periodical arrays with deep ridges has a long history, although they have almost always been considered with an infinite extent in the transverse direction. Gagnepain et al.11 have obtained that such gratings support a family of pure shear surface waves, that grow in numbers as the ridge height increases. Some of us discussed all the families of surface waves that exist for anisotropic and piezoelectric substrates12,13 and obtained their dispersion relations.14 1D phononic crystals were characterized by inelastic light scattering for phonons propagating normally to the ridges15,16 and along an array of corrugated ridges.17 In all these

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works, phonons were efficiently guided at the surface but their transverse extent was not restrained. In contrast, Socié et al.\textsuperscript{18} demonstrated a non-diffracting transducer relying on a locally-resonant waveguide composed of pillars only one wavelength wide. The transducer, however, only operated at the Brillouin zone boundary, for a fixed wavenumber $k = \pi/a$ fixed by the lattice constant $a$.

In this paper, we investigate the dispersion relation for a chain of locally-resonant pillars of sub-wavelength diameter, as depicted in Figure 1. We especially discuss this structure as a waveguide for surface phonons. The infinite chain is found to support non radiating (i.e. non leaky) guided modes. It also supports leaky guided quasi-modes in the radiative region of $k - \omega$ space. The mechanism for guidance is provided by the hybridization of the pillar resonance with the continuum of elastic waves of the substrate.\textsuperscript{19} Simulations performed on a chain of finite size show that the elastic energy can be strongly trapped in the pillars, resulting in an enhancement of the total displacement compared to the one obtained at the homogeneous substrate surface.

II. DISPERSION OF GUIDED SURFACE PHONONS

Figure 2 presents the dispersion relation for elastic waves in a one-dimensional phononic crystal of pillars sitting on a surface. It is well known that acoustic phonons are equivalent to elastic waves in the limit of long wavelengths compared to atomic separations. With this equivalence, we can use continuum mechanics to obtain the phononic band structure. Both the underlying substrate and the pillar are supposed to be made of aluminum. The material properties used in all calculations are mass density $\rho = 2700 \text{ kg/m}^3$, Young’s modulus $E = 70 \text{ GPa}$, and Poisson coefficient $\eta = 0.35$. The periodic unit cell consider for finite element analysis of dispersion is three-dimensional. The wavenumber $k$ is imposed along axis $x$ where the lattice constant is $a_x = a$. A substrate with a finite depth $h = 20a$ along axis $z$ is considered in all band structure computations, with a clamped boundary condition being applied at the bottom face of the finite element mesh. This setting is only an approximation to the semi-infinite problem but the vertical truncation of the mesh does not significantly alter the estimation of the dispersion relation outside of the sound cone of the substrate.\textsuperscript{20} Note that the clamped boundary condition at the bottom face serves to remove the Rayleigh surface wave that would appear at the free bottom surface if it were not applied.

In the case of the linear chain of pillars, the lateral truncation of the computation domain along axis $y$ is also critical because of the possible radiation of wave energy along the surface in the lateral direction. In practice, we considered periodic boundary conditions along that axis, i.e. we considered a super-cell, with the lattice constant $a_y$ adjusted so that the dispersion is not significantly altered. In our numerical experiments, convergence of the dispersion relation was obtained for $a_y/a \geq 5$ for sufficiently high pillars. Beyond this value, we found that the periodic and free boundary conditions give almost the same dispersion relation. Hence the value $a_y = 5a$ is used for band structure computations in the following.

The case of shallow pillars is presented in Figure 2(a). If the ratio $h_p/a$ of the pillar height to the lattice constant is small (here, $h_p/a = 0.1$), only one surface guided mode appears below the sound cone. As a reminder, the sound cone is the part of $k - \omega$ space containing the dispersion of the bulk waves of the substrate or radiative region; any surface guided solution falling in this region would indicate the possible leakage of wave energy. The observed dispersion of the surface guided wave is very close to that of the Rayleigh wave of the substrate, showing a small Bragg band gap opened by periodicity just below the sound cone at the X point of the first Brillouin zone. The displacement...
FIG. 2. Dispersion diagram for guided surface phonons in a infinite and periodic chain of cylindrical pillars sitting on a semi-infinite medium. In the numerical computation, pillars and substrate are supposed to be made of aluminum and $r/a = 0.35$.

(a) The dispersion diagram for shallow pillars ($h_p/a = 0.1$) is obtained from finite element analysis using the mesh shown as an inset. The total displacement for the surface guided wave $R$ is plotted at the X point of the first Brillouin zone.

(b) The dispersion diagram for tall pillars ($h_p/a = 0.6$) is obtained from finite element analysis using the mesh shown as an inset. The total displacement for the surface guided modes $M_1$ to $M_4$ are plotted at the X point of the first Brillouin zone.

field for the Rayleigh Bloch wave is shown in Figure 2(a) with the label $R$. Bragg reflections occur periodically at the pillar locations and the Bragg band gap width is a measure of the reflection coefficient: it equals zero for $h_p/a = 0$ and increases as the pillar height increases.

The case of tall pillars ($h_p/a = 0.6$) is presented in Figure 2(b). There are now more branches of the dispersion appearing under the sound cone. The first four of them are labeled $M_1$ to $M_4$ and are discussed next. The Bragg band gap for surface waves is still present, but is now wider as the reflection coefficient has increased. The representative Rayleigh Bloch wave is $M_4$. The Rayleigh SAW of the substrate further hybridizes with flexural resonances of the pillar to form a system of two bands in the low frequency range. In the absence of coupling with the substrate, the flexural resonances of the pillar would be degenerate in frequency. Bloch wave $M_1$ has its flexural motion in the sagittal plane, $(x,z)$, while Bloch wave $M_2$ has its flexural motion in the transverse plane, $(y,z)$. Owing to propagation
along the x axis and hence to different coupling mechanisms, their hybridization with the Rayleigh surface wave is slightly different: Bloch wave M₁ has a larger coupling strength and hence a stronger dispersion around the resonance. A second ‘breathing’ resonance with displacements mostly in the (x,y) plane creates a very flat band around \( va = 1120 \text{ m/s} \). In this case, there is almost no hybridization with the Rayleigh SAW and Bloch wave M₃ is mostly independent of the wavenumber \( k \). Such a mode of vibration strictly does not exist for \( h_p/a = 0 \).

One striking property of all guided Bloch waves with tall pillars is that their lateral extent is limited to the strict spatial vicinity of the pillar; if they extend over the substrate then it can only be through fast-decaying evanescent tails. This property can be understood mathematically via the hybridization mechanism of a localized resonant vibration in the pillar and the continuum of the elastic waves in the substrate. At frequency \( \omega \), each elastic wave in the continuum can be labeled by its wavevector \( (k_x = k_x, k_y, k_z) \) and the slowness surface to which it belongs. In a general elastic solid, there are exactly 3 slowness surfaces for bulk elastic waves; one is for quasi-longitudinal waves and the other two are for quasi-shear waves. Providing \( k \) makes the dispersion of the guided wave to fall outside of the sound cone, then both \( k_x \) and \( k_z \) have to be complex with a non zero imaginary part: all bulk waves of the substrate appearing in the plane wave spectrum of the guided mode must then be evanescent. As a result, the linear chain of pillars forms a segmented waveguide for surface waves.

The dependence of the various guided modes with the pillar radius was further explored by varying the value of \( r/a \) while keeping the ratio of pillar height to lattice constant fixed to \( h_p/a = 0.6 \), as summarized in Figure 3. Three different values of \( r/a \) are considered in the figure: 0.35, 0.4, and 0.49; for the latter value successive pillars are almost touching and their surface coupling is in principle strongest. The features of the dispersion diagram discussed above are found to be resilient to a change in the value of \( r/a \). The Bragg band gap for the Rayleigh wave is only very slightly affected, indicating that the reflection coefficient does not change appreciably. The hybridized resonances M₁ and M₂ are more affected, with the system of 2 bands becoming less flat as \( r/a \) increases toward 0.5, reflecting the increase in surface coupling between adjacent pillars. The resonance frequency of the isolated pillar for the fundamental flexural mode depends mostly on \( h_p \) rather than on \( r \); hence the amount of repulsion between the two bands is the main factor explaining their increasing separation.

**FIG. 3.** Dependence of the dispersion relation with the ratio of pillar radius to lattice constant, \( r/a \). The considered values of \( r/a \) are (a) 0.35, (b) 0.4, and (c) 0.49. The locally-resonant band gap is shaded in gray in each case.
as \( r/a \) increases. The flat dispersion of Bloch wave \( M_3 \) is not significantly affected but its frequency moves down as \( r/a \) increases. Given the ‘breathing’ nature of this resonance, such a behavior is expected as more matter has to move around during vibration of the pillar.

### III. LEAKAGE IN THE RADIATIVE REGION

So far we have only considered waves that are truly guided at the surface and along the pillar chain. Such true surface waves certainly exist below the sound cone, as we noted already. When the wavenumber-frequency couple \((k, \omega)\) in the dispersion relation is located inside the sound cone, waves propagating at the surface can become coupled to bulk waves radiating away into the depth of the substrate. They are then said to be leaky.

A proper mathematical treatment of leakage for surface phononic crystals remains elusive.\(^4\) Indeed, it would be necessary to take into account properly the semi-infinite nature of a substrate supporting a phononic crystal layer. The semi-infinite geometry can be represented for instance via the full anisotropic Green’s function of the substrate.\(^1,21,22\) The connection of such an approach with the 3D finite element representation of the phononic crystal layer, however, is yet to be implemented. In order to estimate numerically the confinement of Bloch waves, we consider in this paper the total energy stored in a given volume \( V \) as the sum of kinetic energy and elastic (potential) energy

\[
H(V) = K + E = \int_V \frac{1}{2} \rho \omega^2 u_i^* u_i + \int_V \frac{1}{2} u_i^* c_{ijkl} u_k u_l
\]

with \( \rho \) the mass density, \( c_{ijkl} \) the stiffness tensor, \( u_i \) the components of the displacement vector, and \( u_k u_l \) the components of the strain tensor. This quantity is easily calculated at each point of the band diagram from the displacement field of the relevant Bloch wave. The integration volume \( V \) can be chosen at will. Here, we consider three specific choices: integration restricted to the pillar only (volume \( V_1 \)); integration restricted to the pillar plus a substrate layer of thickness \( 2a \) (volume \( V_2 \)); and integration over the full computational domain (volume \( V_0 \), in the case considered obtained for \( h = 20a \)). If the ratio \( H(V_1)/H(V_0) \) is close to 1, then the total energy of the Bloch wave is mostly confined in the pillar. In this case, it can be said that vibrations are mostly within the pillar and that they are likely due to excitation of a resonance. If the ratio \( H(V_2)/H(V_0) \) is close to 1, then the Bloch wave is presumably guided within a small depth below the surface. For the Rayleigh surface wave and for shallow pillars, the wavelength is of the order of \( 2a \) at the X point of the first Brillouin zone, so most of the energy of the wave is contained in volume \( V_2 \). In the case of tall pillars, we keep the same criterion for measuring the penetration depth of surface guided waves. For leaky guided waves, energy is spread in the whole substrate and the energy stored in \( V_2 \) relative to the energy in the whole substrate should in contrast be small.

Figure 4 shows the band diagram of Figure 2(b) with the added information of relative surface stored energy, \( H(V_2)/H(V_0) \) for Figure 4(a) and \( H(V_1)/H(V_0) \) for Figure 4(b). For all surface guided waves, the relative energy inside volume \( V_2 \) is close to one. Surface guided waves appear below the sound cone. The distribution of their energy inside the substrate is exponential and decreases rapidly with depth. More interestingly, it is observed that some dispersion bands appear neatly in the sound cone. These are solutions in \( k - \omega \) space that are not true surface guided modes, i.e. their energy is not confined to the vicinity of the surface, but that are waves whose local density of states at the surface shows a maximum compared to the radiation background. Such leaky waves represent ‘resonances’ of the surface that are only weakly coupled to the radiation continuum. Such waves can be excited by a surface perturbation and can propagate for a certain distance along the surface before they exponentially decay.

When integration of the energy is restricted to the pillar, as shown in Figure 4(b), only the bands corresponding to resonances of the pillars remain. In particular, Bloch wave \( M_3 \) is confirmed to correspond to vibrations that exist mostly inside the pillar and to couple only weakly to bulk waves in the substrate. Hence, this vibration can exist in the sound cone with limited radiation loss.
FIG. 4. Dispersion diagram for guided surface phonons in a infinite and periodic chain of cylindrical pillars sitting on a semi-infinite medium. In the numerical computation, pillars and substrate are supposed to be made of aluminum, \( r/a = 0.35 \), and \( h_p/a = 0.6 \). The normalized total energy stored (a) in a volume \( V_2 \) including the pillars and a substrate layer of thickness \( 2a \), and (b) in a volume \( V_1 \) including only the pillars is shown as color information. Normalization is with respect to the total energy \( H(V_0) \) contained in the full computational domain.

IV. GUIDANCE ALONG A FINITE CHAIN OF PILLARS

Having identified surface phonons suitable for transporting wave energy along an infinite chain of pillars, we now consider the excitation of a finite sequence of 7 pillars. Transmission through the chain is estimated as a function of frequency by placing a line source in the near-field of the first pillar, as depicted in Figure 5. A similar line is placed after the last pillar in order to record arriving surface waves. In comparison to band diagram computations, the periodic boundary conditions have been removed and perfectly matched layer (PML) regions have been added to the four sides of the

FIG. 5. Transmission of surface phonons through a finite chain of 7 pillars. (a) The transmission obtained by finite element analysis is shown for a line source polarized in the sagittal plane, with displacements in the plane \( (u_x, u_z) \), and for a pure shear line source, with displacements \( u_y \). (b) Total displacement distribution for \( fa = 1120 \) m/s.
computation domain. In addition, another PML region is added at the bottom of the computation domain. The PML regions effectively absorb all waves radiating away from the line source and the pillars, both laterally and in the depth of the substrate.

The transmission shown in Figure 5(a) is defined as the total displacement at the receiving line relative to the total displacement at the source line. Two polarizations are considered for the source, with displacements either in the sagittal plane, \((u_x, u_y)\), or transverse shear \((u_z)\). Since none of the surface guided phonons are of a pure polarization, either sources can in principle excite all of them.

The main maximum transmission with the sagittal source, however, clearly appears at the frequency of Bloch wave \(M_2\) \((fa = 640\, m/s)\). Two main transmission maxima are observed with the transverse shear source, at the frequencies of Bloch waves \(M_1\) \((fa = 640\, m/s)\) and \(M_3\) \((fa = 1120\, m/s)\). As a note, the transmission spectra around \(fa = 640\, m/s\) have a typical Fano-like shape, as often observed with local resonances.

Figure 5(b) depicts the distribution of the total displacement, in the pillars and at the surface, for the sagittal source and at the frequency of Bloch wave \(M_2\). Displacements in the pillars are clearly enhanced compared to displacements at the source line. Since the dispersion of the guided surface phonon is very flat, its group velocity is rather low and Rayleigh surface waves of the free surface can hardly couple into the waveguide. Once they have entered the waveguide, however, they remain effectively trapped, causing an enhancement of the total displacement. For the same reason, the line source we considered is not a priori well suited for efficient coupling to the waveguide. Enhanced source design is left for future work.

V. CONCLUSION

We have considered the propagation of surface elastic waves, or surface phonons, along a linear and periodic chain of cylindrical pillars sitting on a semi-infinite solid substrate. A variety of guided modes, some of them exhibiting a very low group velocity, have been identified. Their frequencies are close to resonance frequencies of the pillars. Although the pillar diameter is typically smaller than half the relevant wavelength, lateral radiation on the surface is found to be canceled by periodicity. Surface guidance is explained by the hybridization of the resonating pillars with the continuum of elastic waves of the substrate. With regards to applications, we remark that the consideration of micro or nanomechanical resonators sitting on a semi-infinite substrate and their mutual coupling is a topic of great interest nowadays, with implications e.g. in frequency control, sensing and optomechanics.

ACKNOWLEDGMENTS

Financial support by the Agence Nationale de la Recherche under grant ANR-14-CE26-0003-01-PHOREST and through Labex ACTION ANR-11-LABX-0001-01 is gratefully acknowledged.

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